CHAPTER 3

IMPEDANCE MATCHING

DEFINITION OF MATCHING (1/2)

The active power provided by a source represented by its Thevenin equivalent, of internal impedance $Z_S \equiv R_S + jX_S$ with a load impedance $Z_L \equiv R_L + jX_L$ is given by:

$$P = R_L |I|^2 = R_L \left(\frac{V_S}{|Z_S + Z_L|} \right)^2 = \frac{R_L V_S^2}{(R_S + R_L)^2 + (X_S + X_L)^2} \tag{3.1}$$

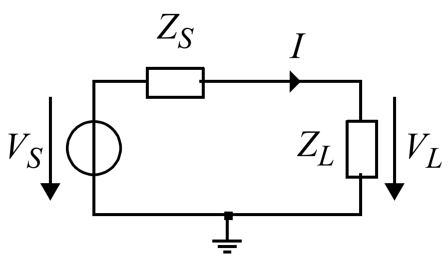


Fig 3-1: *Source connected to a load.*

For a given source impedance, the power dissipated in R_L is maximum when:

$$\frac{\partial P}{\partial X_L} = 0 \Rightarrow X_L = -X_S$$

$$\frac{\partial P}{\partial R_L} = 0 \Rightarrow R_S^2 - R_L^2 + (X_S + X_L)^2 = 0 \Rightarrow R_L = R_S \tag{3.2}$$

and thus when:

$$\boxed{Z_L = Z_S^*} \tag{3.3}$$

The power transfer from a source to a load is therefore maximum when the load impedance is equal to the complex conjugate of the source impedance. This situation corresponds to *matching*.

DEFINITION OF MATCHING (2/2)

When the load impedance is matched to the source impedance, the load reactance has the opposite sign from the source reactance, and thus they mutually compensate each other. The resulting circuit corresponds to a series connection of the source and load resistances, which are equal, permitting a maximum transfer of power from the source to the load. If the source reactance is that of an inductance, the load reactance should be that of a capacitance, and vice versa. This matching is only valid at the resonance frequency of the inductance and capacitance in series.

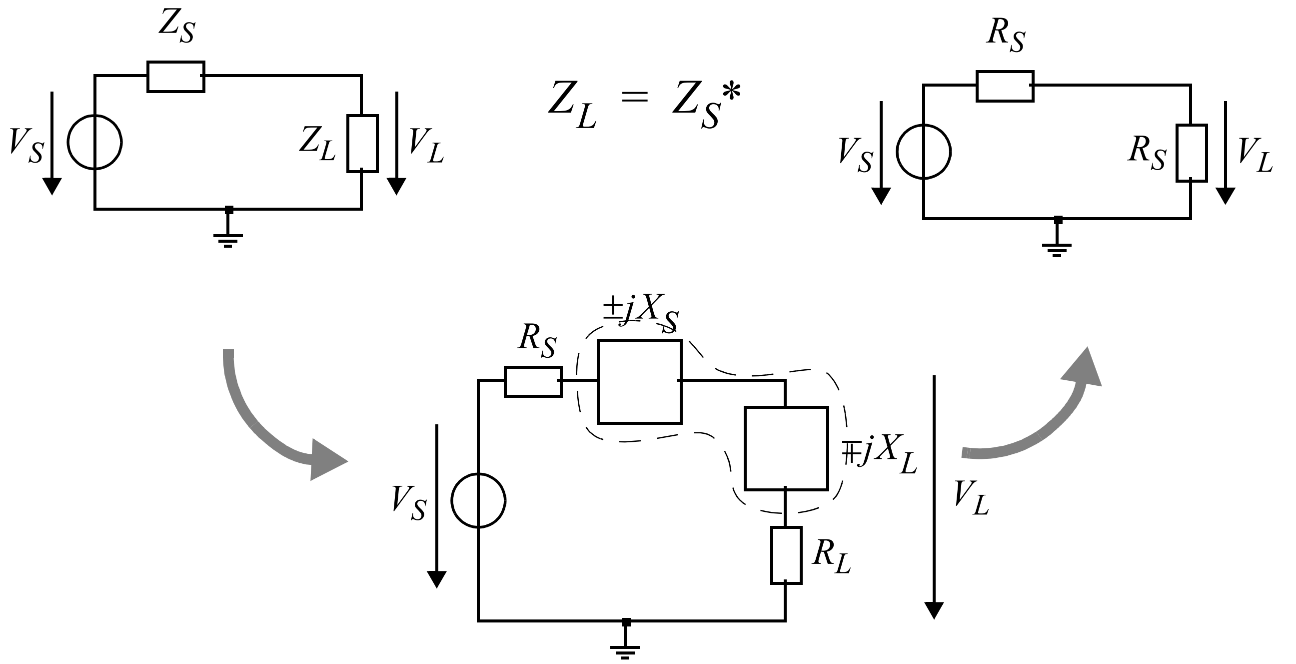


Fig 3-2: *Matching the load.*

Impedance matching is thus only strictly realized at a single frequency, which is the resonant frequency of the series resonant circuit. The matching worsens as the frequency gets farther from the resonant frequency, which can cause problems for circuits with a large passband. There are methods for increasing the band of frequencies for which there is matching and therefore maximum power transfer. These methods generally use circuits with a low quality factor.

PRINCIPLE OF IMPEDANCE MATCHING

Impedance matching consists of synthesizing a non-dissipating circuit (thus containing only inductors and capacitors) inserted between the source and the load, such that the impedance as seen from the source is equal to the complex conjugate of the source impedance. Of course there are an infinite number of circuits, more or less complex, that satisfy this criterion.

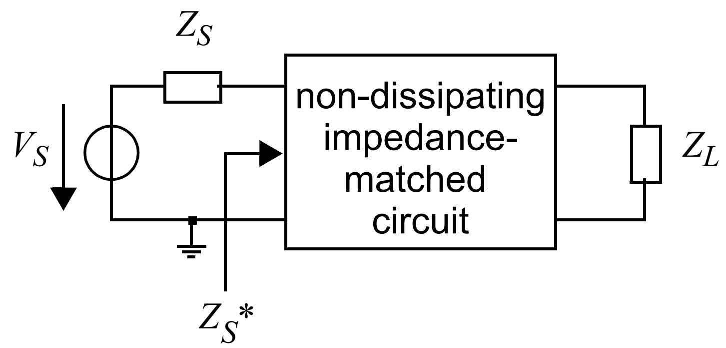
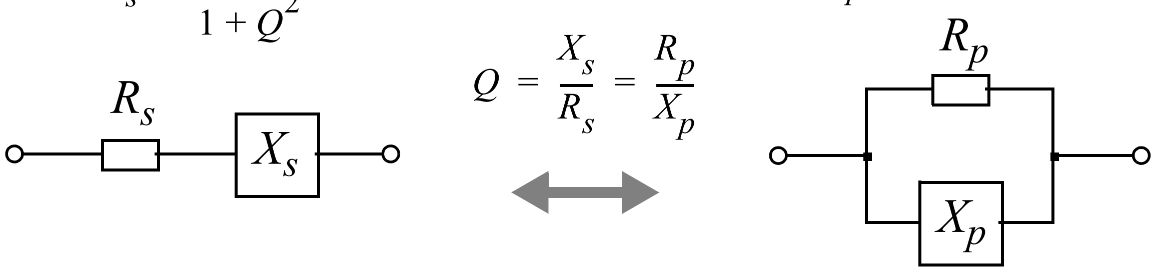


Fig 3-3: *Principle of impedance matching.*

All of the following methods are based on the equivalence between the series and parallel circuits illustrated in Fig. 3-4. The unloaded Q is the quality factor of the impedance-matched circuit associated either with the source or the load resistance. The loaded Q is the quality factor of the complete circuit (with the source and load). Since $R_S = R_L$, the loaded quality factor is equal to half of the unloaded quality factor.

$$R_s = \frac{R_p}{1+Q^2} \qquad R_p = R_s(1+Q^2)$$

$$Q = \frac{X_s}{R_s} = \frac{R_p}{X_p}$$



$$X_s = QR_s = \frac{X_p}{1+1/Q^2} \qquad X_p = \frac{R_p}{Q} = X_s(1+1/Q^2)$$

Fig 3-4: *Series / parallel equivalence.*

L NETWORKS (1/4)

When the source and load impedances are purely resistive and $R_L > R_S$, it is necessary to lower the impedance seen from the source by placing a reactance (inductor or capacitor) in parallel with the load. One must then compensate the reactance of the shunt element just added by placing a reactance with the opposite sign in series. In the case shown in Fig. 3-5 there is a capacitor C in parallel with the load R_L .

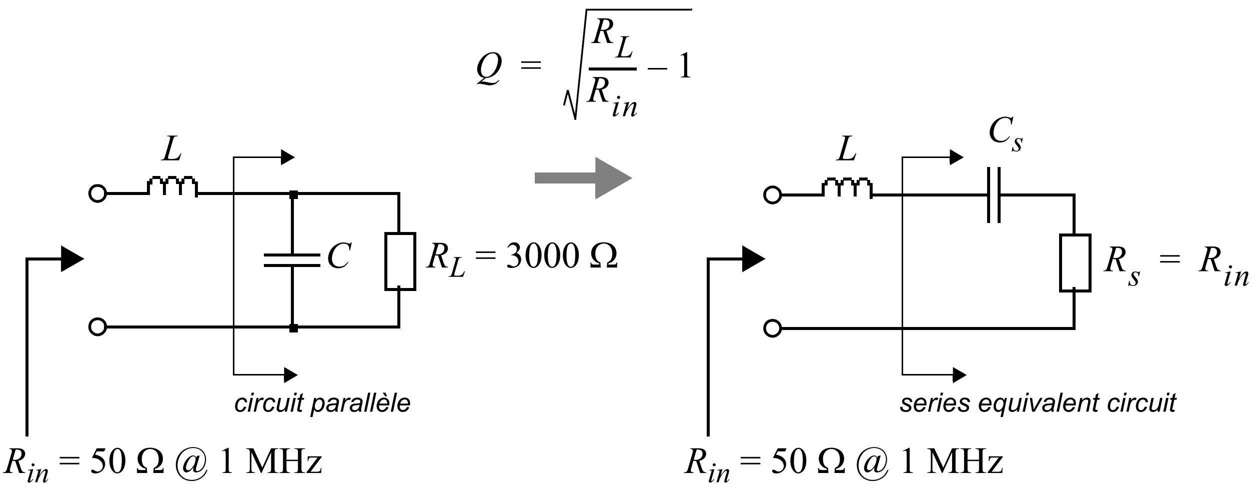


Fig 3-5: *Example of an L circuit, impedance step-down.*

This parallel circuit can be transformed to an equivalent series circuit in which the series resistance will be equal to the source resistance, in this case 50 Ω. The two have the same quality factor:

$$R_s = \frac{R_L}{1+Q^2} \Rightarrow Q = \sqrt{\frac{R_L}{R_s} - 1} = \sqrt{\frac{3000}{50} - 1} = 7.7 \qquad (3.4)$$

from which we calculate the reactance of the inductor and the capacitor:

$$X_L = X_{Cs} = QR_s = QR_{in} = 7.7 \times 50 = 384\Omega$$

$$X_C = R_L / Q = 3000/7.7 = 391\Omega$$

and thus:

$$L = X_L/(2\pi f) = 384/(2\pi \times 10^6) = 69\mu H$$

$$C = 1/(2\pi f X_C) = 1/(2\pi \times 10^6 \times 391) = 407.5pF$$

L NETWORKS (2/4)

The amplitude of the impedance of the circuit in Fig. 3-5 is represented in Fig. 3-6. We note that the reactance of Z_{in} cancels out at the resonant frequency for which the input impedance is equal to 50 Ω. Notice that the circuit in Fig. 3-5 performs low-pass filtering. In certain cases, a high-pass characteristic is preferable, and is obtained by simply switching the capacitor and the inductor.

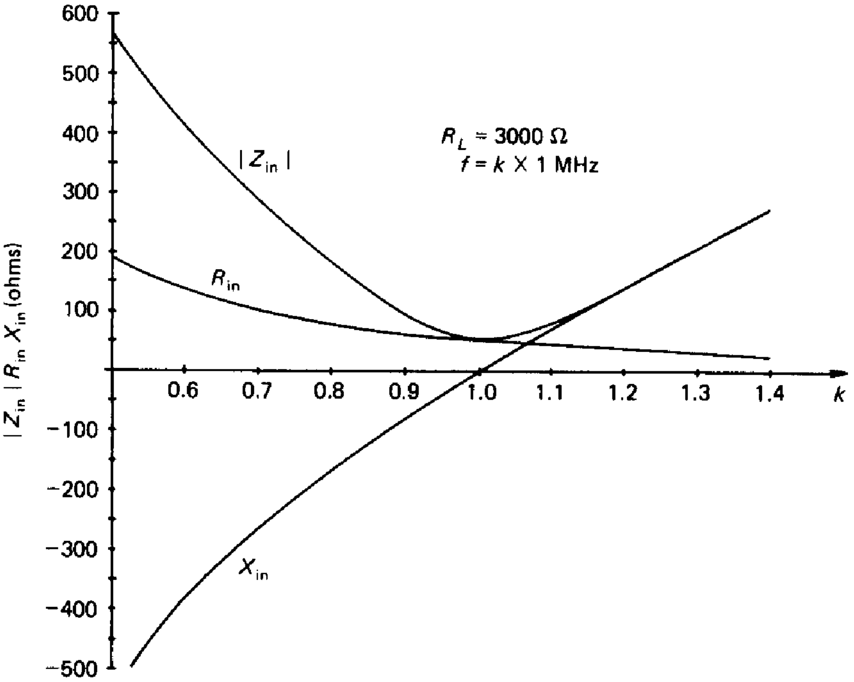
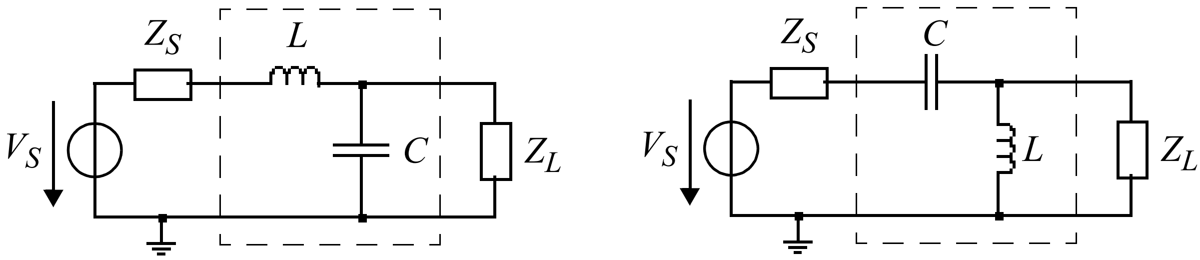


Fig 3-6: *Input impedance of the circuit in Fig. 3-5.*

Since the two reactances must be of opposite signs, one of the components will be an inductor and the other a capacitor. There are therefore only two L networks which lower the impedance as seen from the source. They are shown in Fig. 3-7.



a) Low-pass filter.　　*b) High-pass filter.*

Fig 3-7: *L networks, impedance step-down ($R_L > R_S$).*

L NETWORKS (3/4)

When the source and load impedances are purely resistive and $R_L < R_S$, it is necessary to increase the impedance seen from the source by placing a reactance (inductor or capacitor) in series with the load. One must then compensate the reactance of the series element just added by placing a reactance of opposite sign in parallel. In the case shown in Fig. 3-8, an inductor L has been placed in series with the load R_L .

$$Q = \sqrt{\frac{R_{in}}{R_L} - 1}$$

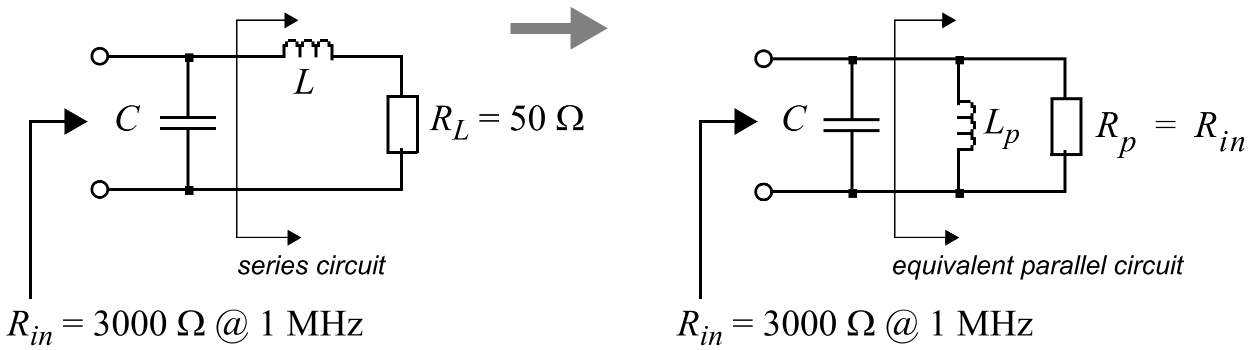


Fig 3-8: *Example of L network, impedance step-up.*

The series circuit can be transformed to its parallel equivalent according to:

$$R_p = R_L(1 + Q^2) \Rightarrow Q = \sqrt{\frac{R_p}{R_L} - 1} = \sqrt{\frac{R_{in}}{R_L} - 1} = 7.7 \qquad (3.5)$$

One can thus calculate the reactance of the inductor and the capacitor:

$$X_C = X_{Lp} = R_p/Q = R_{in}/Q = 3000/7.7 = 391\Omega$$

$$X_L = QR_L = 7.7 \times 50 = 384\Omega$$

and the values of the components for the desired frequency:

$$C = 1/(2\pi f X_C) = 1/(2\pi \times 10^6 \times 391) = 407.5pF$$

$$L = X_L/(2\pi f) = 384/(2\pi \times 10^6) = 61.1\mu H$$

L NETWORKS (4/4)

The amplitude of the impedance of the circuit in Fig. 3-8 is represented in Fig. 3-6. We note once again that the reactance of Z_{in} cancels out at the resonant frequency for which the input impedance is equal to 3000 Ω. The circuit in Fig. 3-8 performs low-pass filtering, which can be changed to high-pass filtering by simply switching the inductor and the capacitor.

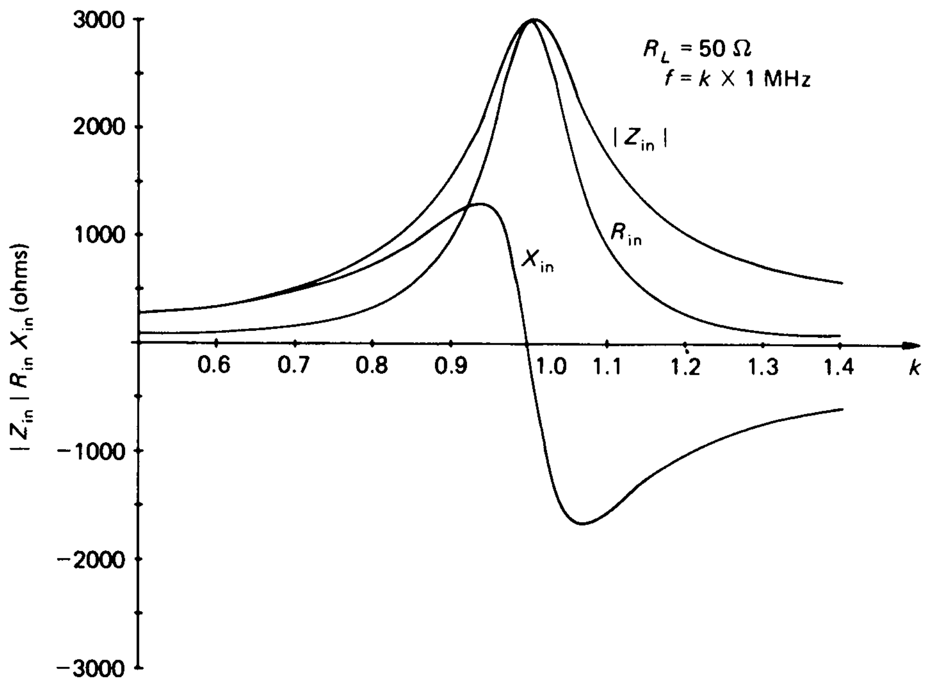
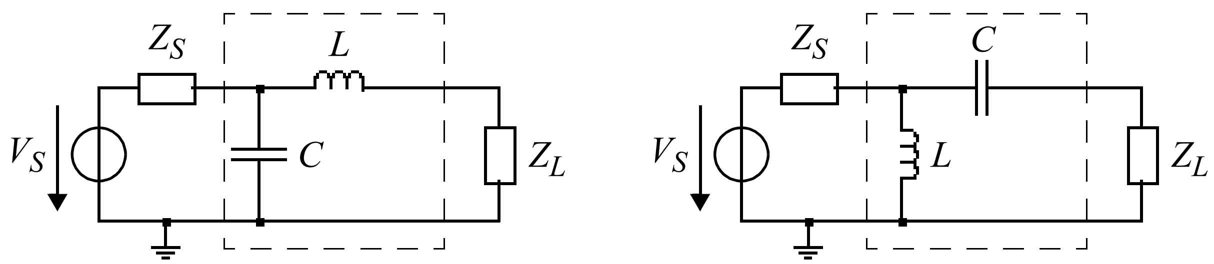


Fig 3-9: *Input impedance for the circuit in Fig. 3-8.*

There are two L networks which allow us to increase the impedance as seen from the source. They are shown in Fig. 3-10.



a) Low-pass filter. *b) High-pass filter.*

Fig 3-10: *L networks, impedance step-up ($R_L < R_S$).*

COMPLEX SOURCE AND LOAD IMPEDANCES (1/4)

In the preceding examples, we supposed that the source and load impedances were real. In reality, they are rarely real. For example, the input and output impedances of a bipolar transistor are always complex. There are two methods for handling the reactances of the source and load:

a) *absorption:* the reactances of the source and load can be taken into account in the impedance matching network by placing the components such that the functional capacitors of the network are in parallel with the parasitic capacitances and the functional inductors in series with the parasitic inductances.

b) *resonance:* cancel out the effect of the reactances of the source and load by placing a reactance of the opposite sign in parallel or in series.

Note that absorption is only possible when the value of the parasitic element is smaller than that of the functional element from which it must be subtracted. These two techniques can naturally be combined.

By way of example we synthesize an impedance matching network using the absorption method for the circuit in Fig. 3-11.

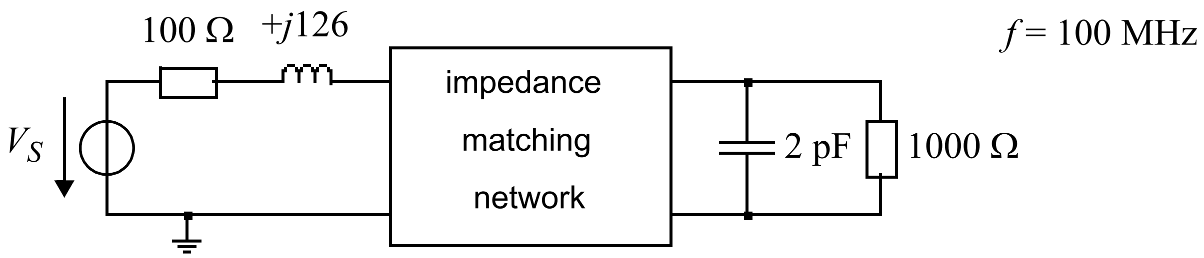


Fig 3-11: *Complex source and load impedances.*

At first we ignore the source and load reactances. The load resistance being larger than the source resistance, we choose the L network of Fig. 3-7 a).

COMPLEX SOURCE AND LOAD IMPEDANCES (2/4)

The quality factor is given by:

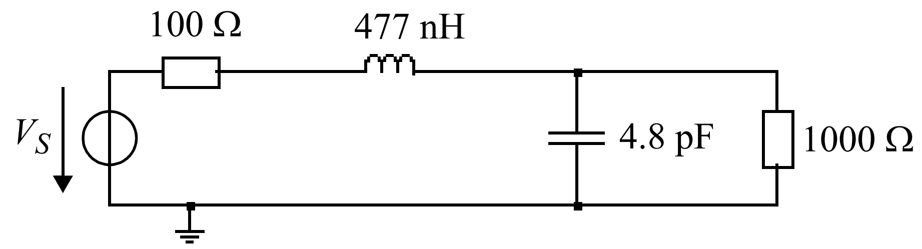
$$Q = \sqrt{R_L/R_S - 1} = 3$$

from which we get:

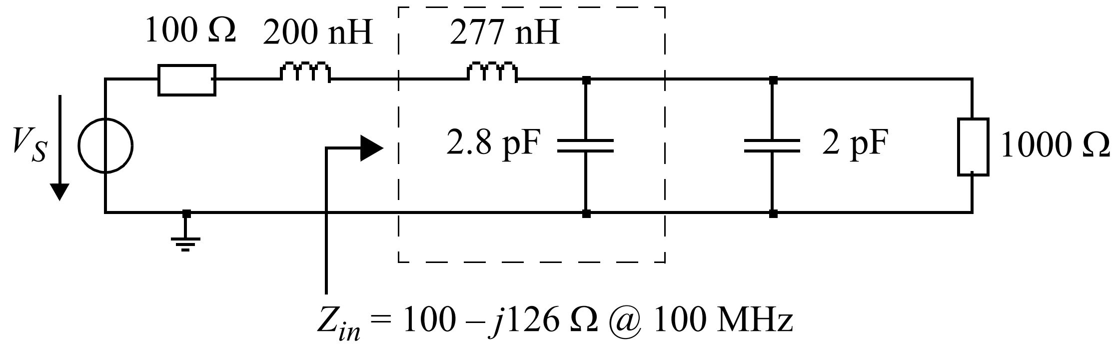
$$X_s = QR_S = 3 \times 100 = 300\Omega \Rightarrow L = \frac{X_s}{2\pi f} = \frac{300}{2\pi \times 10^8} = 477nH$$

$$X_p = \frac{R_p}{Q} = \frac{R_L}{Q} = \frac{1000}{3} = 333\Omega \Rightarrow C = \frac{1}{2\pi f X_p} = \frac{1}{2\pi \times 10^8 \times 333} = 4.8pF$$

We thus obtain the diagram shown in Fig. 3-12 a). By then subtracting the value of the 477nH series source inductance and the value of the 4.8 pF parallel load capacitance, we obtain the diagram in Fig. 3-12 b).



a)



b)

Fig 3-12: *Illustration of the absorption method.*

COMPLEX SOURCE AND LOAD IMPEDANCES (3/4)

Another example, illustrating the load resonance technique, is given in Fig. 3-13 a). We want to synthesize a high-pass impedance-matching circuit. The fact that the load resistance is larger than the source resistance means that we must use the circuit shown in Fig. 3-7 b). But before calculating the elements of the L network, we must rid ourselves of the load capacitance by connecting a parallel inductor whose value is calculated according to:

$$L = \frac{1}{(2\pi f)^2 C_L} = \frac{1}{(2\pi \times 75\times10^6)^2 \times 40\times10^{-12}} = 112.6nH$$

We therefore get the circuit of Fig. 3-13 b) from which we can calculate the elements of the L network:

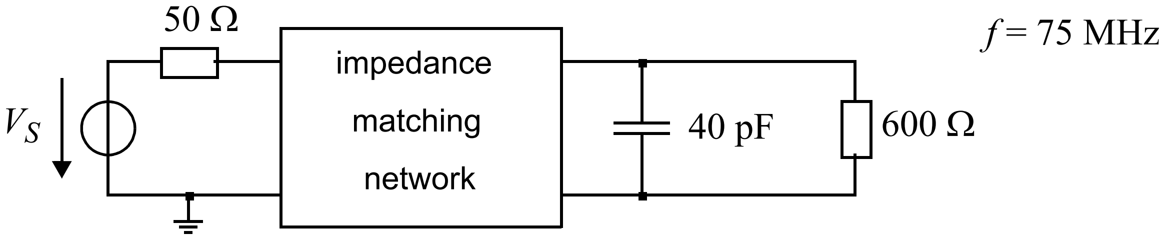
$$Q = \sqrt{\frac{R_L}{R_S} - 1} = \sqrt{\frac{600}{50} - 1} = 3.32$$

$$X_s = QR_S = 3.32 \times 50 = 166\Omega \Rightarrow C = \frac{1}{2\pi f X_s} = \frac{1}{2\pi \times 75\times10^6 \times 166} = 12.78pF$$

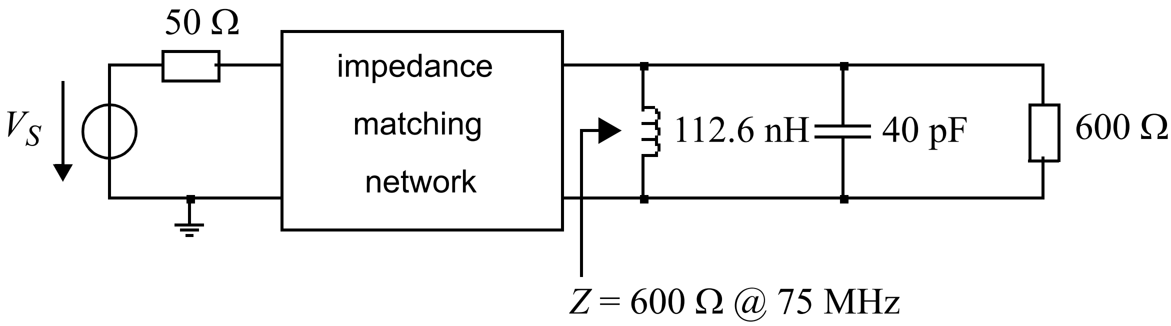
$$X_p = \frac{R_p}{Q} = \frac{R_L}{Q} = \frac{600}{3.32} = 181\Omega \Rightarrow L = \frac{X_p}{2\pi f} = \frac{181}{2\pi \times 75\times10^6} = 384nH$$

We thus obtain the circuit in Fig. 3-13 c) which can be simplified further by calculating the equivalent inductance for the two parallel inductances connected to the load. Finally, we obtain the circuit in Fig. 3-13 d).

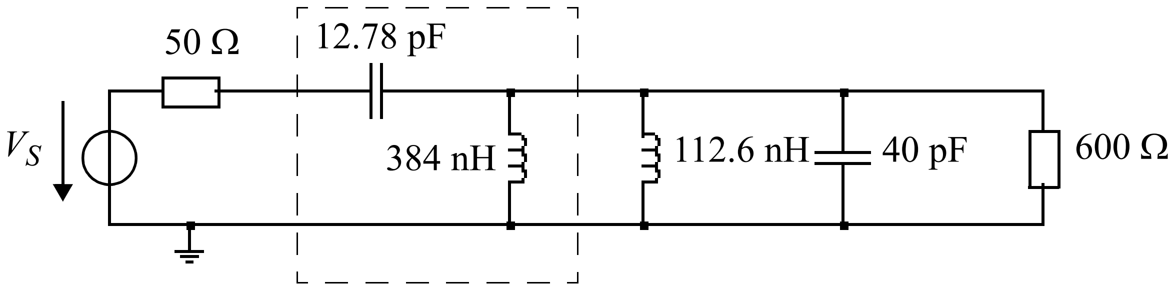
COMPLEX SOURCE AND LOAD IMPEDANCES (4/4)



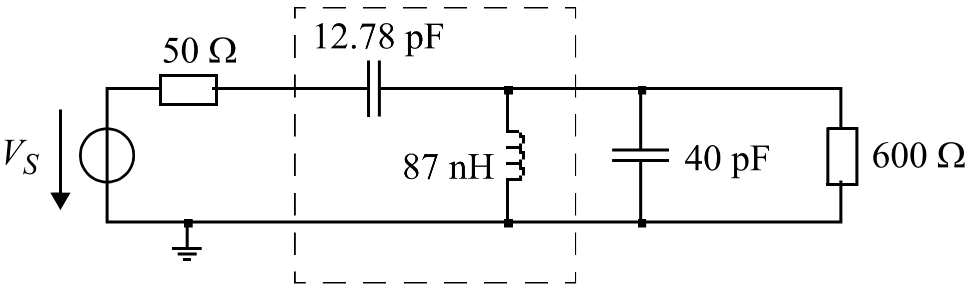
a) Circuit with complex load.



b) Addition of an inductor to compensate the load capacitance by resonance.



c) Synthesis of an L network for a resistive load.



d) Final circuit.

Fig 3-13: *Illustration of the resonance technique.*

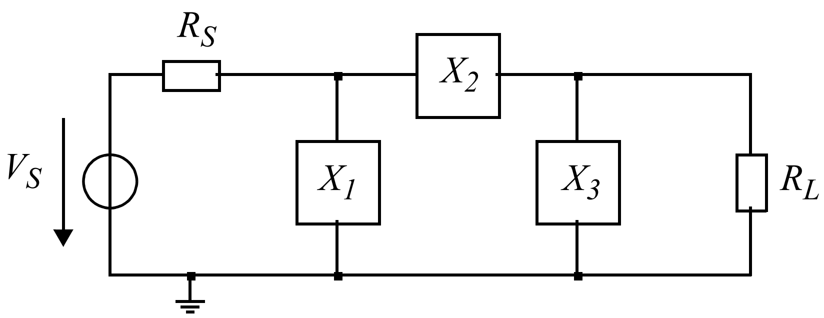
THREE-ELEMENT MATCHING NETWORKS

The disavantage of L networks comes from the fact that when the source and load resistances are specified, the quality factor and therefore the selectivity of the impedance-matching network are likewise specified (cf Eqn. 3.4 and 3.5). There are then not enough degrees of freedom to choose the quality factor independently, which can be irritating for certain applications in which we want selectivity. To compensate for this problem, it is possible to add an element and thus a degree of freedom permitting us to set the quality factor. This Q will necessarily be larger than the quality factor corresponding to an L network. The L network is thus the impedance-matching network having the minimum quality factor.

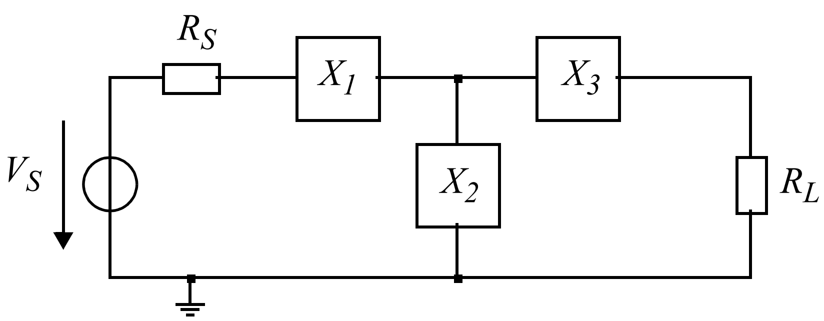
There are two types of three-element matching networks (cf Fig. 3-14):

1) Π networks;

2) T networks;



a) Π *network.*



b) T *network.*

Fig 3-14: *3-element matching networks.*

Π NETWORKS

One can describe the Π network as the connection of two L networks with a virtual resistance as shown in the diagram in Fig. 3-15. This virtual resistance is just used to dimension the elements of the L networks. The reactances X_{s1} and X_{p1} as well as X_{s2} and X_{p2} must be of different types (if for example X_{s1} corresponds to a capacitance, X_{p1} must correspond to an inductance).

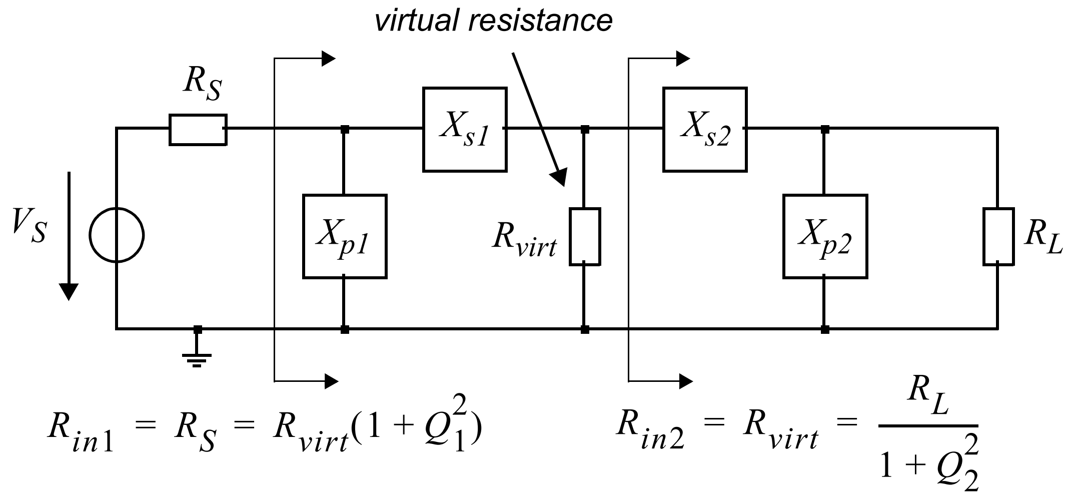


Fig 3-15: Π *network represented as two L networks.*

The virtual resistance R_{virt} represents the resistance seen from the center point $R_{in2} = R_{virt} = R_L/(1+Q_2^2)$ with $Q_2 = R_L/X_{p2}$, from which we get $Q_2 = \sqrt{R_L/R_{virt}-1}$. In addition, the resistance seen from the source must be equal to R_S, imposing $R_{in1} = R_S = R_{virt}(1+Q_1^2)$ with $Q_1 = X_{s1}/R_{virt}$ and therefore $Q_1 = \sqrt{R_S/R_{virt}-1}$. We remark that for the quality factors Q_1 and Q_2 to exist, the virtual resistance must be less than R_S or R_L.

The quality factor of the Π network is associated to that of the L network section having the larger quality factor, and the section having the larger quality factor is on the side with the higher terminating impedance. This gives us the definition of the quality factor of a Π network:

$$\boxed{Q_\Pi \equiv \sqrt{\frac{R_{max}}{R_{virt}}-1}} \tag{3.6}$$

where R_{max} represents the larger of the resistances R_S or R_L.

Π NETWORK EXAMPLE (1/2)

As an example, we will match a load resistance of 50 Ω to a source resistance of 3000 Ω by using a Π network, conserving a quality factor of 10 (cf Fig. 3-16).

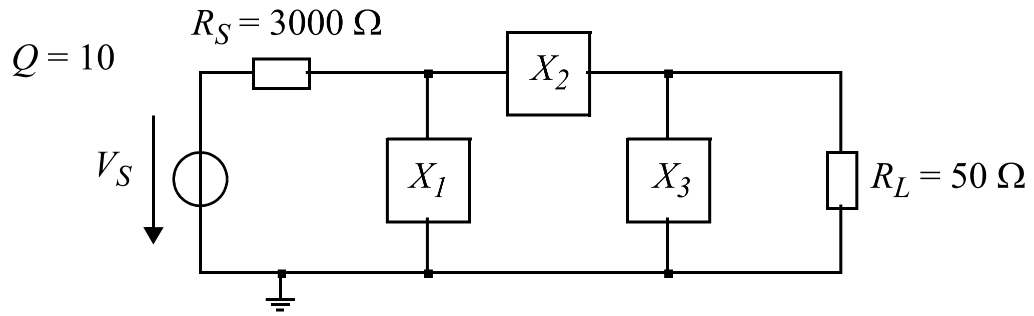


Fig 3-16: *Example of designing a Π network.*

The frequency is equal to 1 MHz. R_{virt} is calculated from Eqn. 3.6 with $R_{max} = R_S = 3000\ \Omega$:

$$R_{virt} = \frac{R_{max}}{1+Q^2} = \frac{3000}{101} = 29.703\Omega$$

The reactances of the first section are thus given by:

$$X_{p1} = R_S/Q = 3000/10 = 300\Omega$$

$$X_{s1} = QR_{virt} = 10 \times 29.7 = 297.03\Omega$$

The quality factor of the second L network section is then set by the resistances R_{virt} and R_L:

$$Q_2 = \sqrt{\frac{R_L}{R_{virt}} - 1} = \sqrt{\frac{50}{29.703} - 1} = 0.8266$$

The resistance R_L must now be matched to the virtual resistance. Since it appears in a parallel branch, we have:

$$X_{s2} = Q_2 R_{virt} = 0.8266 \times 29.7 = 24.55\Omega$$

$$X_{p2} = R_L/Q_2 = 50/0.8266 = 60.49\Omega$$

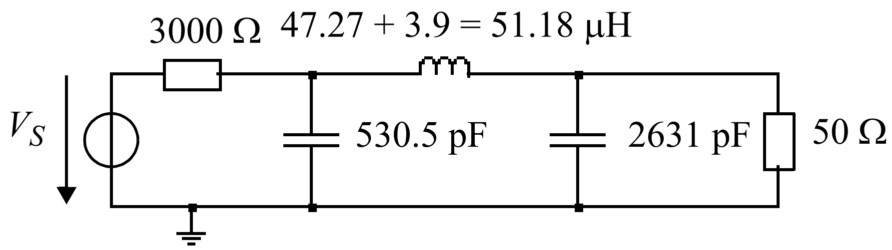
As a result of choosing inductors for the series branches, the shunt branches will therefore be capacitors:

$$C_1 = \frac{1}{2\pi f X_{p1}} = 530.5pF \qquad L_1 = X_{s1}/(2\pi f) = 47.27\mu H$$

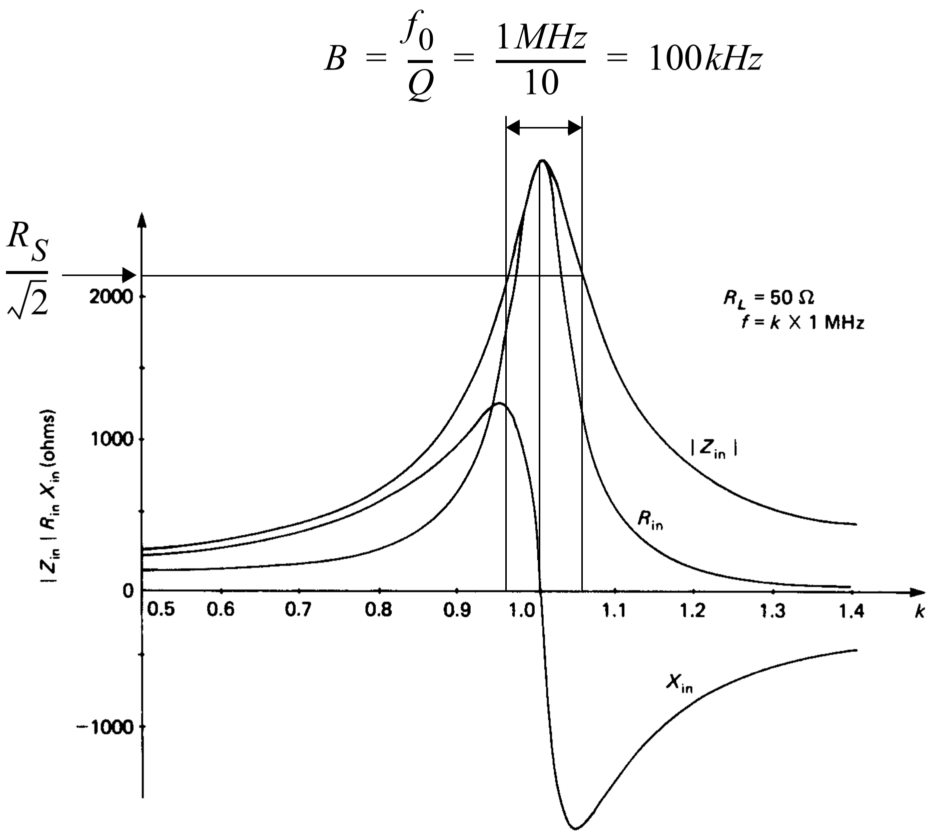
$$C_2 = \frac{1}{2\pi f X_{p2}} = 2631pF \qquad L_2 = X_{s2}/(2\pi f) = 3.9\mu H$$

Π NETWORK EXAMPLE (2/2)

We finally obtain the circuit shown in Fig. 3-17 a), for which the magnitude of the input impedance is shown as a function of the frequency in Fig. 3-17 b). We notice that the imposed quality factor corresponds well to the bandwidth at -3 dB and that the form factor is larger than that of Fig. 3-6, because this is a 3rd order filter.



a) Final circuit.



b) Input impedance of the circuit in Fig. 3-17 a).

Fig 3-17: *Example of the design of a Π network.*

T NETWORKS

The T network can be described as two back-to-back L networks of which the shunt branches are in parallel, as shown in Fig. 3-18. The difference with respect to the Π network is that in the T network, the virtual resistance is larger than both the source and load resistances. The T network is often used for matching small impedances with a high quality factor.

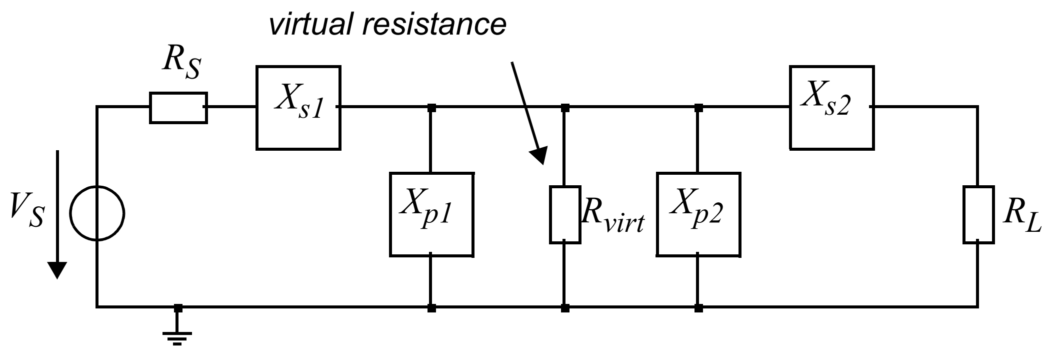


Fig 3-18: *The T network represented as two L networks.*

The quality factor of the T network is determined by the L network section with the higher quality factor. By definition, the section with the higher quality factor is at the side with the smaller terminating resistor. Q is determined by the formula:

$$\boxed{Q_T \equiv \sqrt{\frac{R_{virt}}{R_{min}} - 1}} \qquad (3.7)$$

where R_{min} is the smaller terminating resistor.

T NETWORK EXAMPLE

As an example, we would like to design a T network to match a source resistance of 10 Ω to a load resistance of 50 Ω with a quality factor of 10. We would like to use a minimum number of inductors, and we want the resulting filter to be of type pass-band.

The virtual resistance is calculated from Eqn. 3.7:

$$R_{virt} = R_{min}(Q^2+1) = R_S(Q^2+1) = 10 \times 101 = 1010\Omega$$

The section with the higher quality factor is on the source side. The reactances of the corresponding L network section are:

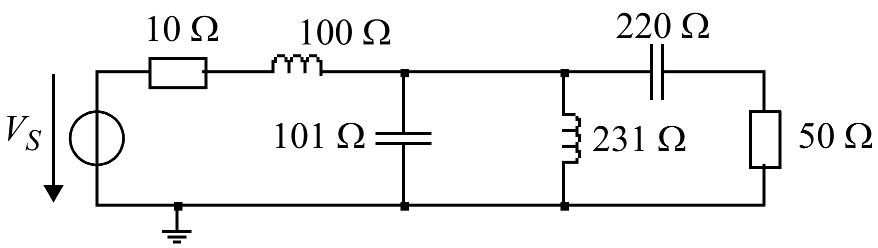
$$X_{s1} = QR_S = 10 \times 10 = 100\Omega \qquad X_{p1} = R_{virt}/Q = 1010/10 = 101\Omega$$

The quality factor of the L network section on the load side is determined by the resistances R_{virt} and R_L:

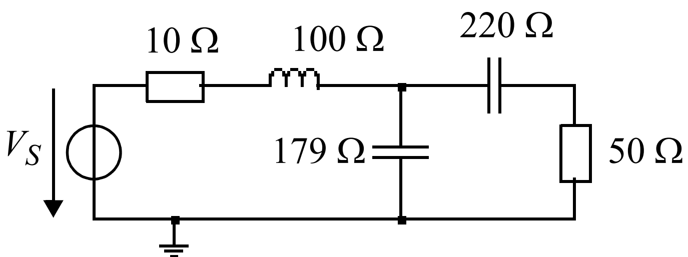
$$Q_2 = \sqrt{R_{virt}/R_L - 1} = \sqrt{1010/50-1} = 4.4$$

$$X_{p2} = R_{virt}/Q_2 = 1010/4.4 = 230\Omega \qquad X_{s2} = Q_2 R_L = 4.4 \times 50 = 220\Omega$$

One possible design in which there is only one inductor and the filtering characteristic is of type passband is shown in Fig. 3-19.



a)

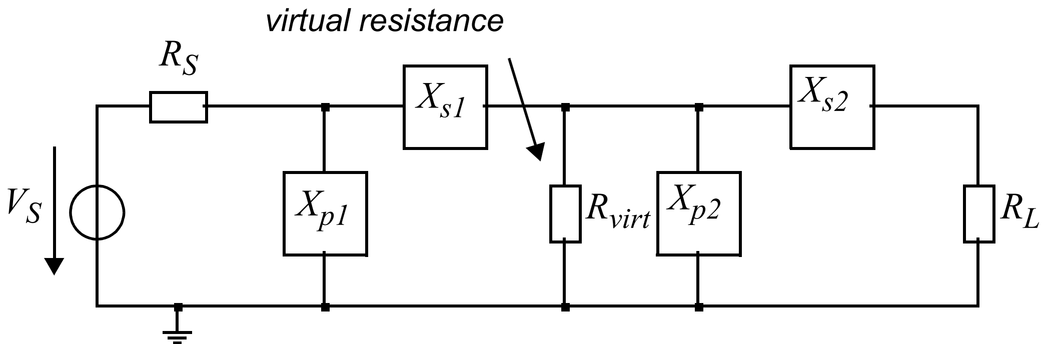


b)

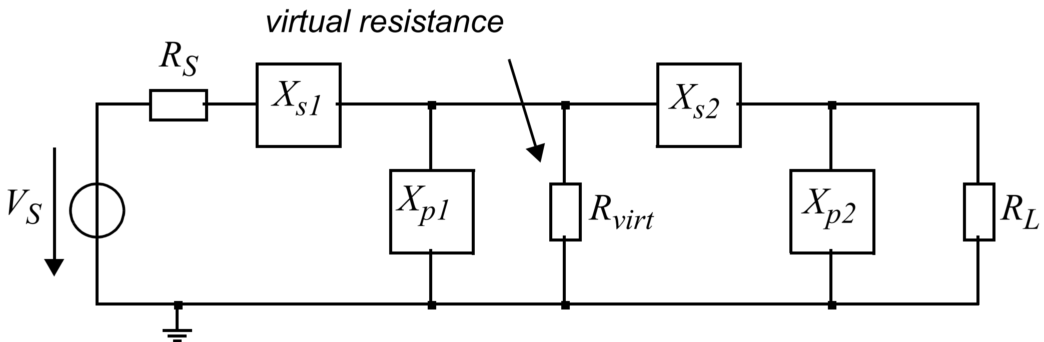
Fig 3-19: *Example of* T *network.*

WIDEBAND IMPEDANCE MATCHING (1/2)

Up until now we have seen L networks for which the quality factor was determined by the source and load resistances, and Π and T networks which allow us to choose a quality factor independently of the source and load, as long as it is higher than that of the L network. These circuits are thus appropriate for narrow-band impedance matching. To match impedances over a wider band (or to have a quality factor smaller than that of the simple L network), we can use two cascaded L networks like those presented in Fig. 3-20. In these configurations, the value of the virtual resistance must be between those of the termination resistances, with the result that the quality factor goes from that of an L network to that of a Π or T network.



a) It can be proved that R_L must be smaller than R_S to use this configuration.



b) It can be proved that R_L must be larger than R_S to use this configuration.

Fig 3-20: *Low quality factor (wideband) matching network.*

WIDEBAND IMPEDANCE MATCHING (2/2)

The minimum quality factor and therefore the maximum band-width are obtained when:

$$R_{virt} = \sqrt{R_S R_L} \tag{3.8}$$

The quality factor is thus defined by:

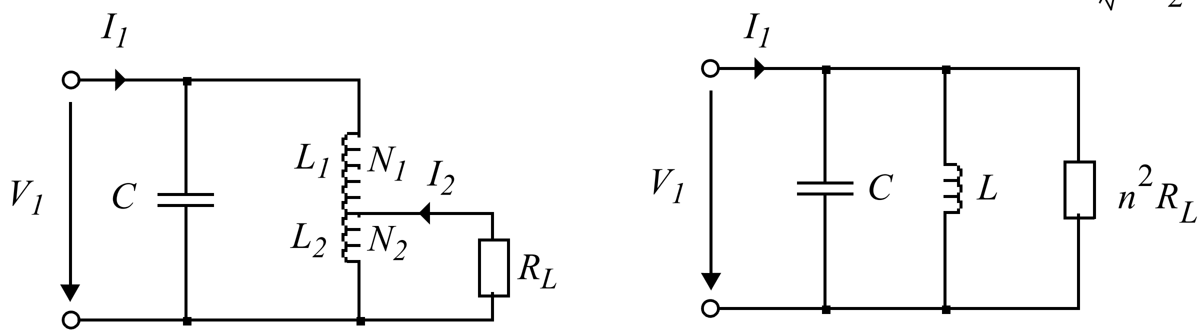
$$Q \equiv \sqrt{\frac{R_{virt}}{R_{min}} - 1} = \sqrt{\frac{R_{max}}{R_{virt}} - 1} \tag{3.9}$$

where R_{virt} is the virtual resistance and R_{min} and R_{max} are, respectively, the smaller and larger terminating resistances.

MATCHING WITH AN AUTOTRANSFORMER

The impedance matching of two circuits can also be carried out by using an inductor with a central lead (or autotransformer) or a capacitive divider. These matching networks are useful when one wants to create, for example, a parallel resonant circuit with a high quality factor, loaded with a small impedance.

$$L = L_1 + L_2 + 2M \qquad k \equiv \frac{M + L_2}{\sqrt{LL_2}}$$



$$n = k \cdot \sqrt{\frac{L}{L_2}} \cong \sqrt{\frac{L}{L_2}}$$

a) Autotransformer. *b) Equivalent circuit.*

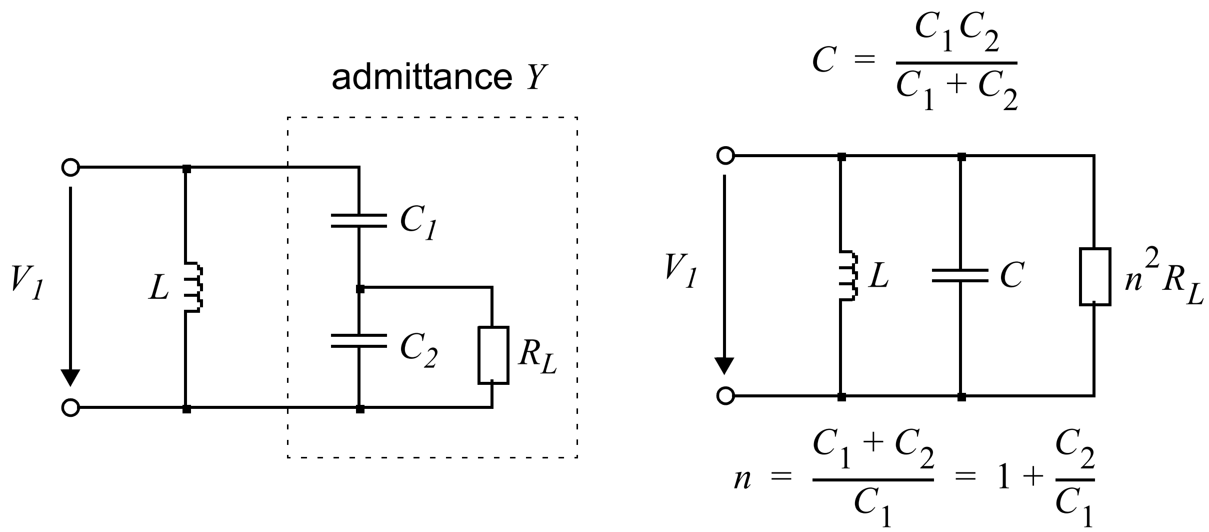
Fig 3-21: *Impedance matching by autotransformer.*

The equivalent resistance in parallel with the LC circuit is equal to the load resistance R_L multiplied by a factor n^2:

$$\boxed{R'_L = n^2 \cdot R_L \cong \frac{L}{L_2} \cdot R_L} \qquad (3.10)$$

IMPEDANCE MATCHING WITH A CAPACITIVE DIVIDER

It is also possible to do impedance matching with a capacitive divider, as shown in Fig. 3-22.



a) Capacitive divider. *b) Equivalent circuit.*

Fig 3-22: *Impedance matching with a capacitive divider.*

The admittance Y appearing in parallel with the inductance L in the diagram in Fig. 3-22 a) has the value:

$$Y(s) = sC_1 \cdot \frac{1 + sR_L C_2}{1 + sR_L(C_1 + C_2)} = G_p + j \cdot X_p \tag{3.11}$$

For frequencies $\omega >> (R_L C_2)^{-1} > [R_L(C_1 + C_2)]^{-1}$, this admittance can be broken up into a parallel conductance

$$G_p \equiv \frac{1}{R'_L} = \frac{(\omega C_1)^2 R_L}{1 + (\omega R_L(C_1 + C_2))^2} \cong \frac{1}{R_L[1 + C_2/C_1]^2} \tag{3.12}$$

and a capacitance C equal to the series connection of C_1 and C_2. The resistance seen at the terminals of the circuit at the resonant frequency of the parallel LC is thus equal to the load resistance multiplied by a factor n^2:

$$\boxed{R'_L = n^2 \cdot R_L = \left[1 + \frac{C_2}{C_1}\right]^2 \cdot R_L} \tag{3.13}$$

SMITH CHARTS

The *Smith Chart* is probably one of the most useful graphical tools for the conception of HF circuits, and specifically for the synthesis of impedance matching networks. It was invented in the 1930's by an engineer at Bell Labs named Phillip Smith. The Smith Chart is a bilinear transformation of the plane of normalized impedances z to the plane of the reflection coefficient Γ:

$$\boxed{\Gamma = \frac{z-1}{z+1} = \frac{Z/Z_0-1}{Z/Z_0+1} = \frac{Z-Z_0}{Z+Z_0}} \tag{3.14}$$

where Z_0 is the normalization impedance, usually 50 Ω. The Smith Chart lets us find the impedance z when we know the reflection coefficient Γ or vice versa.

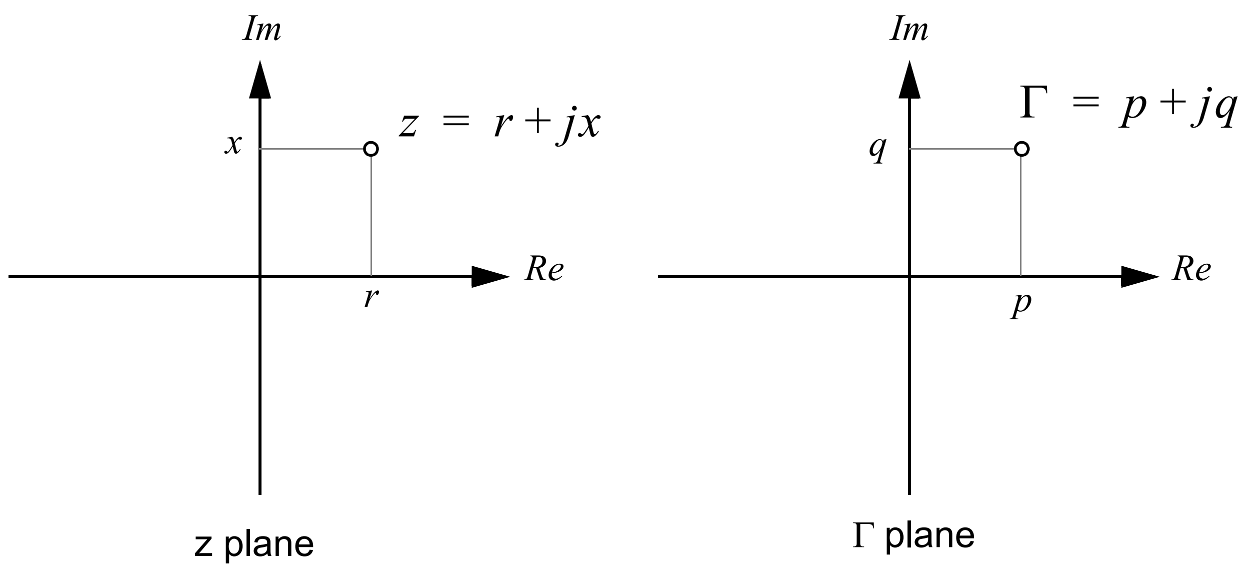


Fig 3-23: *Transformation of the z plane to the Γ plane.*

By setting $z = r + jx$ and $\Gamma = p + jq$, and knowing r and x, p and q must be determined from the following relationship:

$$\Gamma = p + jq = \frac{(r-1)+jx}{(r+1)+jx} \tag{3.15}$$

SMITH CHART CONSTRUCTION (1/2)

Constant resistance circles

By setting the real and imaginary parts of Eqn. 3.15 to be equal, we find the equations which describe the curves of constant r:

$$\left(p - \frac{r}{r+1}\right)^2 + q^2 = \left(\frac{1}{r+1}\right)^2 \qquad (3.16)$$

as well as those of constant x:

$$(p-1)^2 + \left(q - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \qquad (3.17)$$

The curves for $r = const$ defined by Eqn. 3.16 are circles with radius $1/(r+1)$ of which the center is located on the real axis at the point $r/(r+1)$. The two intersections with the real axis are located at $(r-1)/(r+1)$ and *1*. For r varying from 0 to 10, we obtain the network of circles shown in Fig. 3-24. Each point on one of these circles has the same (normalized) resistance.

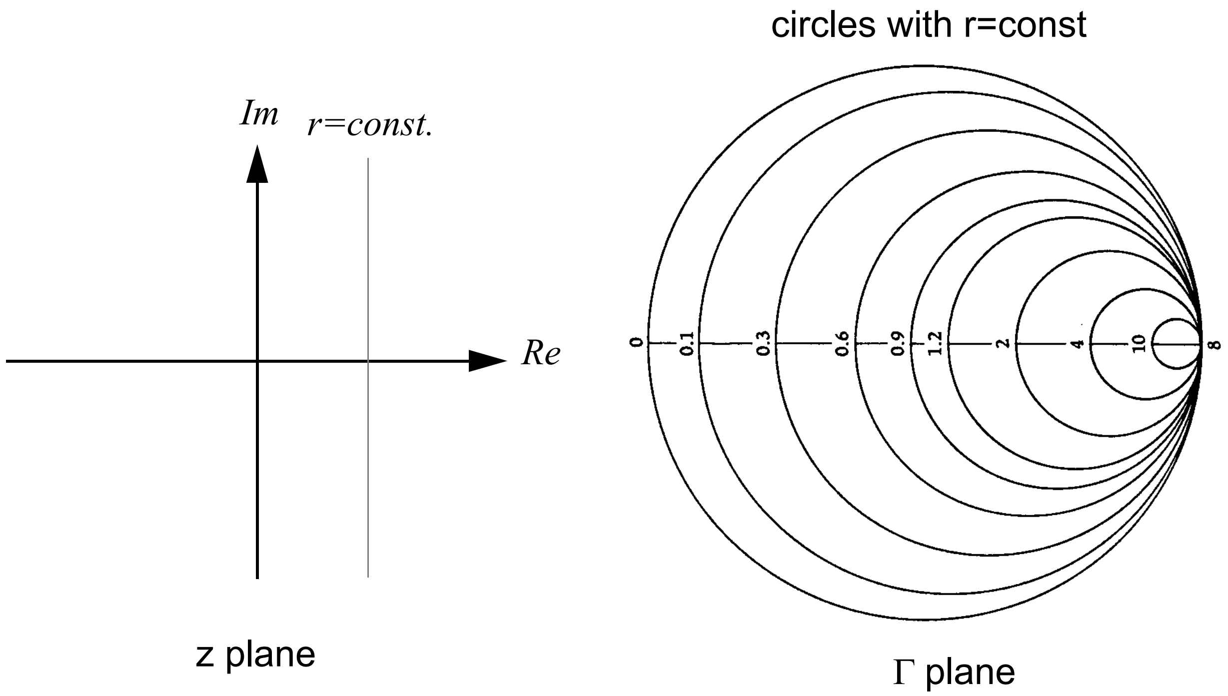


Fig 3-24: *Constant resistance circles*

SMITH CHART CONSTRUCTION (2/2)

Constant reactance circles

The curves with $x = const$ defined by Eqn. 3.17 are also circles, with radius $1/x$ of which the center is located at coordinates $(1, 1/x)$. For x varying from 0.1 to 10, we get the network of circles shown in Fig. 3-25. Each point of one of these circles has the same (normalized) reactance.

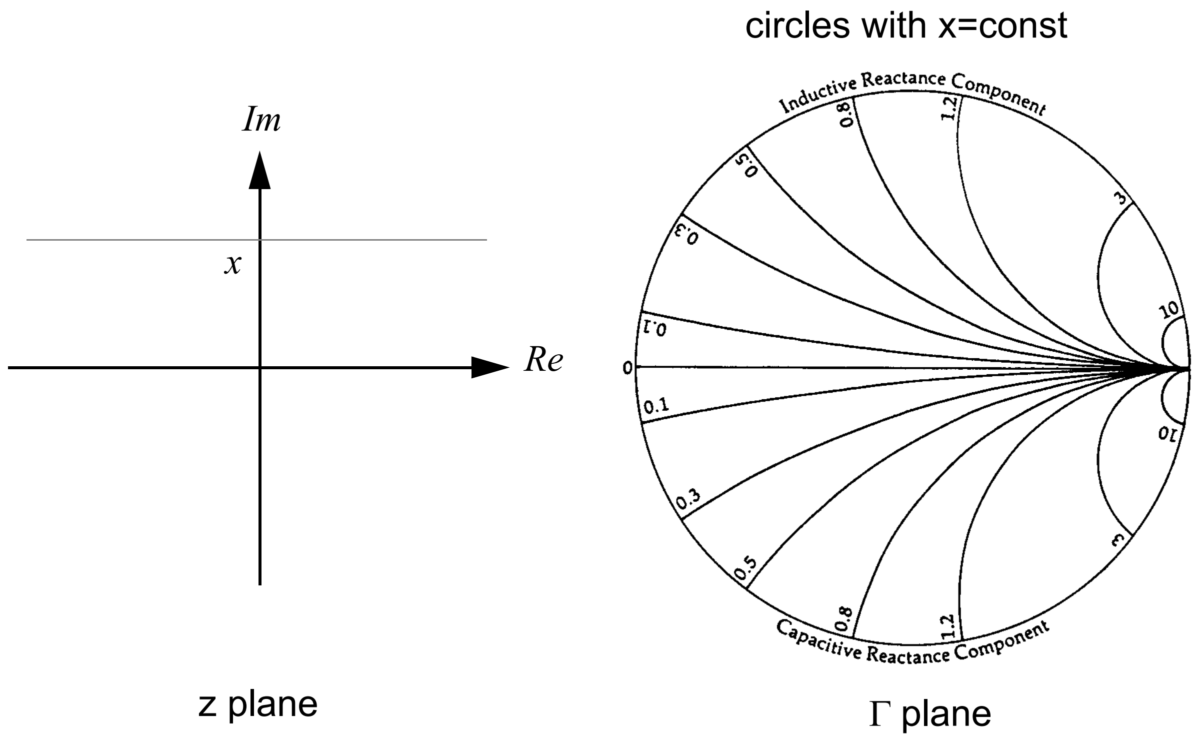


Fig 3-25: *Constant reactance circles.*

IMPEDANCE CHARTS

The superposition of constant resistance and constant reactance circles gives the complete Smith Chart of impedances, as shown in Fig. 3-26. The exterior circle corresponds to zero resistance or a purely imaginary impedance. The upper part corresponds to a positive reactance and thus to an inductance, while the lower part corresponds to a negative reactance and thus to a capacitance. The horizontal diameter corresponds to zero reactance and thus to a purely resistive impedance.

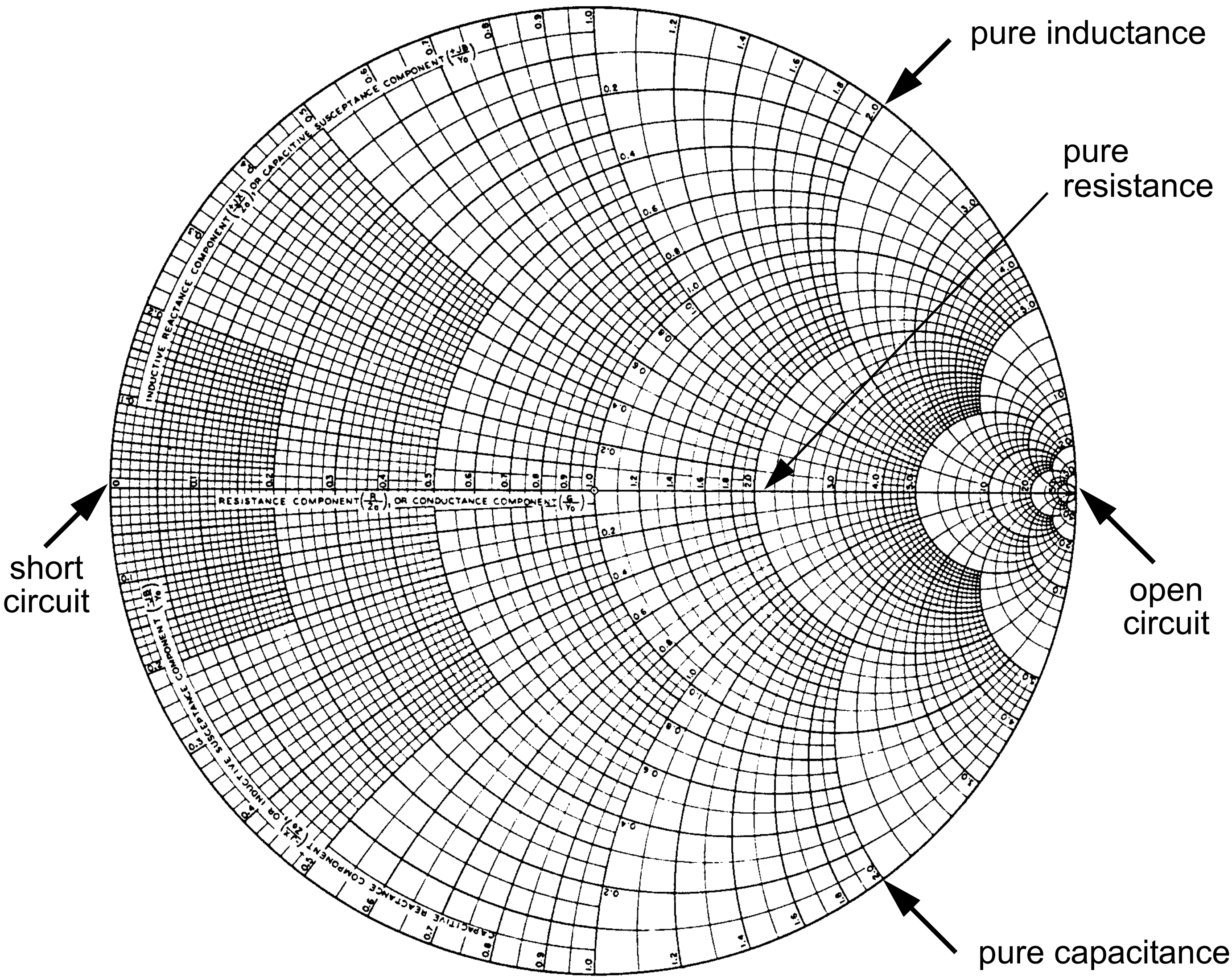


Fig 3-26: *Impedance chart.*

ADDITION OF A SERIES CAPACITOR

Fig. 3-27 represents the effect of the series addition of a normalized negative reactance $-j1.0$ (corresponding to a capacitance) with a normalized impedance $z = 0.5 + j0.7$. The resulting impedance is thus given by $z = 0.5 + j0.7 - j1.0 = 0.5 - j0.3$. The series addition of this capacitor corresponds graphically to moving around the constant resistance circle $r = 0.5$ counter-clockwise.

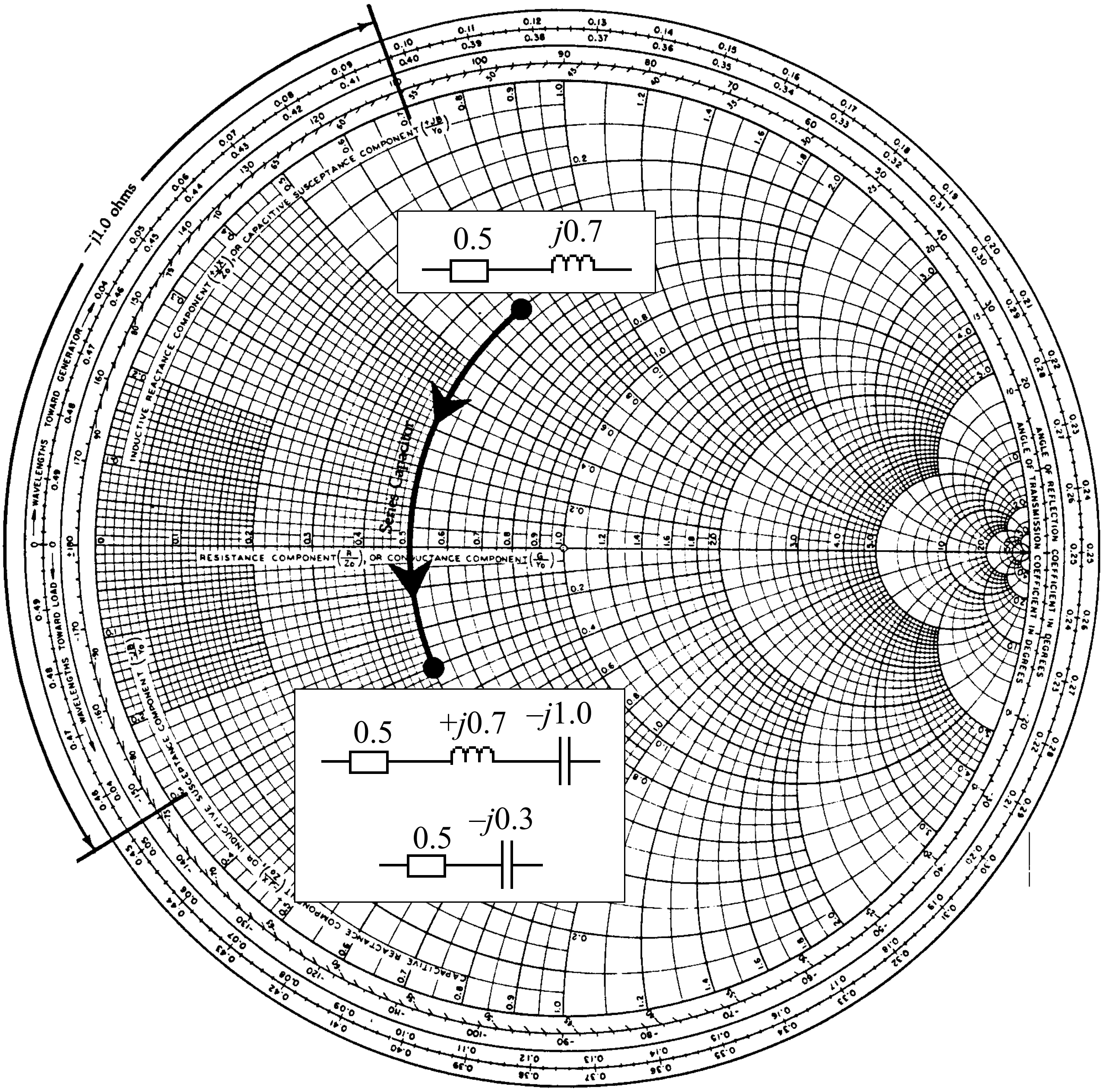


Fig 3-27: *Addition of a series capacitor.*

ADDITION OF A SERIES INDUCTOR

Fig. 3-28 represents the effect of the series addition of a positive reactance $j1.8$ (corresponding to an inductance) with a normalized impedance $z = 0.8 - j1.0$. The resulting impedance is equal to $z = 0.8 - j1.0 + j1.8 = 0.8 + j0.8$. The series addition of this inductor corresponds graphically to moving around the constant resistance circle 0.8 clockwise.

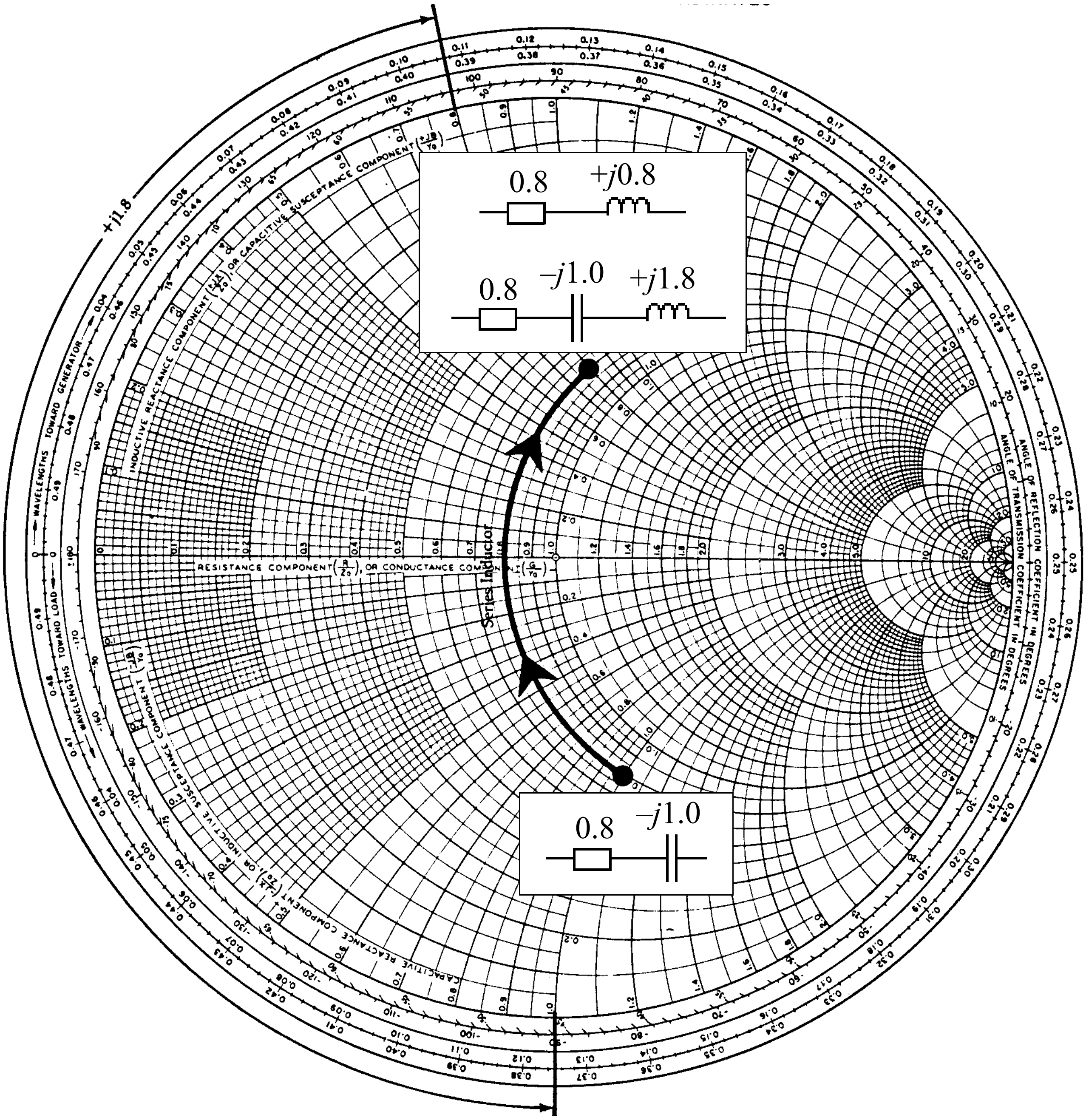


Fig 3-28: *Addition of a series inductor.*

CONVERTING IMPEDANCE TO ADMITTANCE

The Smith Chart can be used to convert an impedance z to an admittance $y = 1/z = g \pm jb$. Let's look at $z = 1 + j$, for example. The corresponding admittance is $y = 1/z = 0.5 - j0.5$. The two corresponding points are shown in Fig. 3-29. Note that they are the same distance d from the origin, but in opposite directions. On the Smith Chart, one easily finds the admittance corresponding to an impedance by moving the distance between z and the origin, but in the opposite direction.

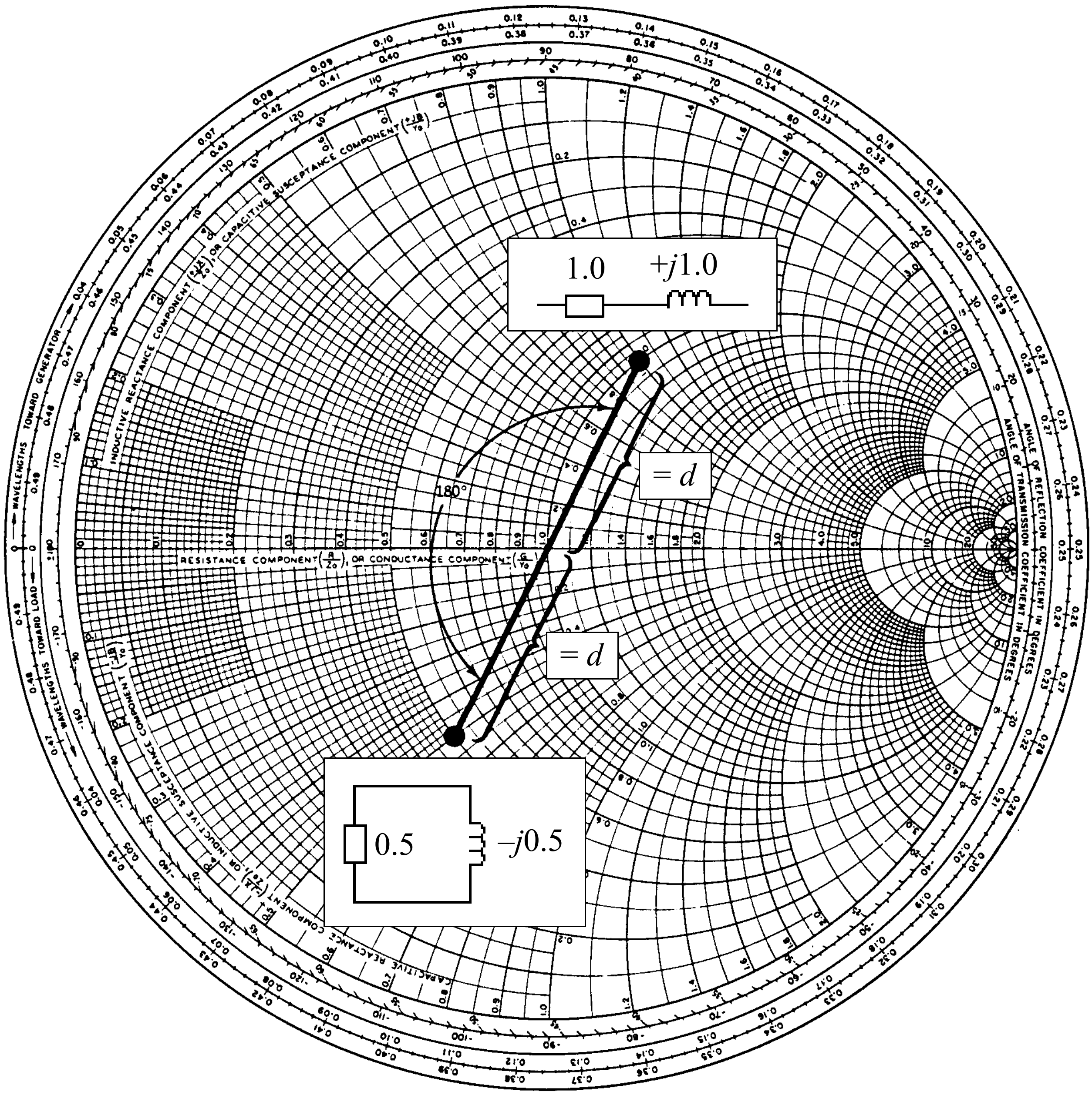


Fig 3-29: *Conversion of impedance to admittance.*

COMBINED IMPEDANCE AND ADMITTANCE CHART

By rotating the impedance chart by 180°, we obtain the admittance chart. Fig. 3-30 shows the superposition of these two charts. One single point now simultaneously corresponds to an impedance and its admittance, of which the values can be read from the respective charts. Notice that because the admittance chart is found by the 180° rotation of the impedance chart, the upper half corresponds to negative susceptances (inductances) and the lower half to positive susceptances (capacitances).

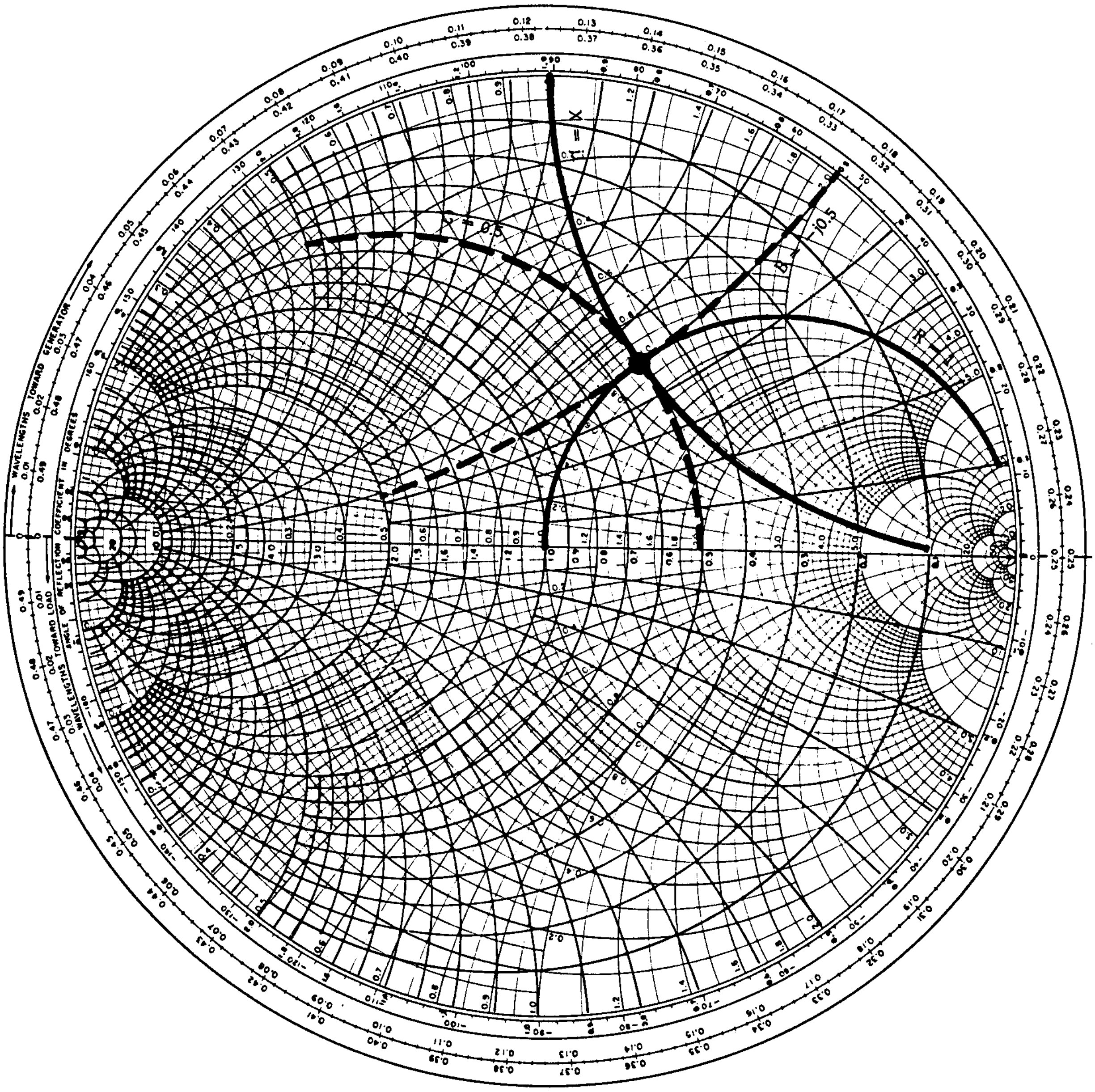


Fig 3-30: *The complete Smith chart.*

ADDITION OF A SHUNT CAPACITOR

Fig. 3-31 shows the effect of the series addition of a positive susceptance $+j0.8$ (capacitance) to an admittance of $y = 0.2 - j0.5$, resulting in an admittance $y = 0.2 + j0.3$. From a graphical point of view, the parallel addition of a capacitor corresponds to moving around a constant conductance circle clockwise.

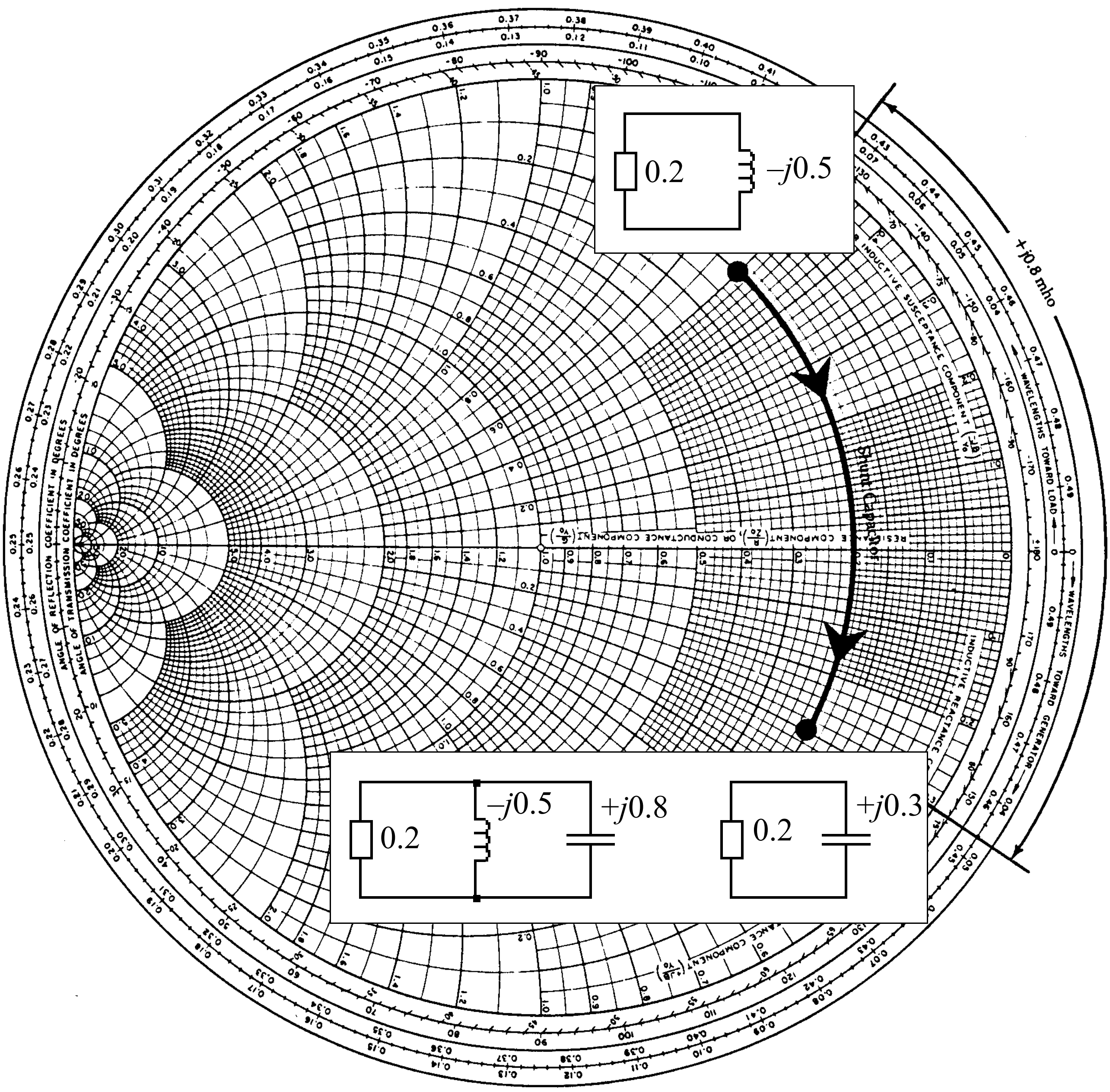


Fig 3-31: *Addition of a shunt capacitor.*

ADDITION OF A SHUNT INDUCTOR

Fig. 3-32 shows the effect of the parallel addition of a negative susceptance $-j1.5$ (inductance) to an admittance $y = 0.7 + j0.8$. The resulting admittance is $y = 0.7 - j$. This operation corresponds graphically to moving around a constant conductance circle counter-clockwise.

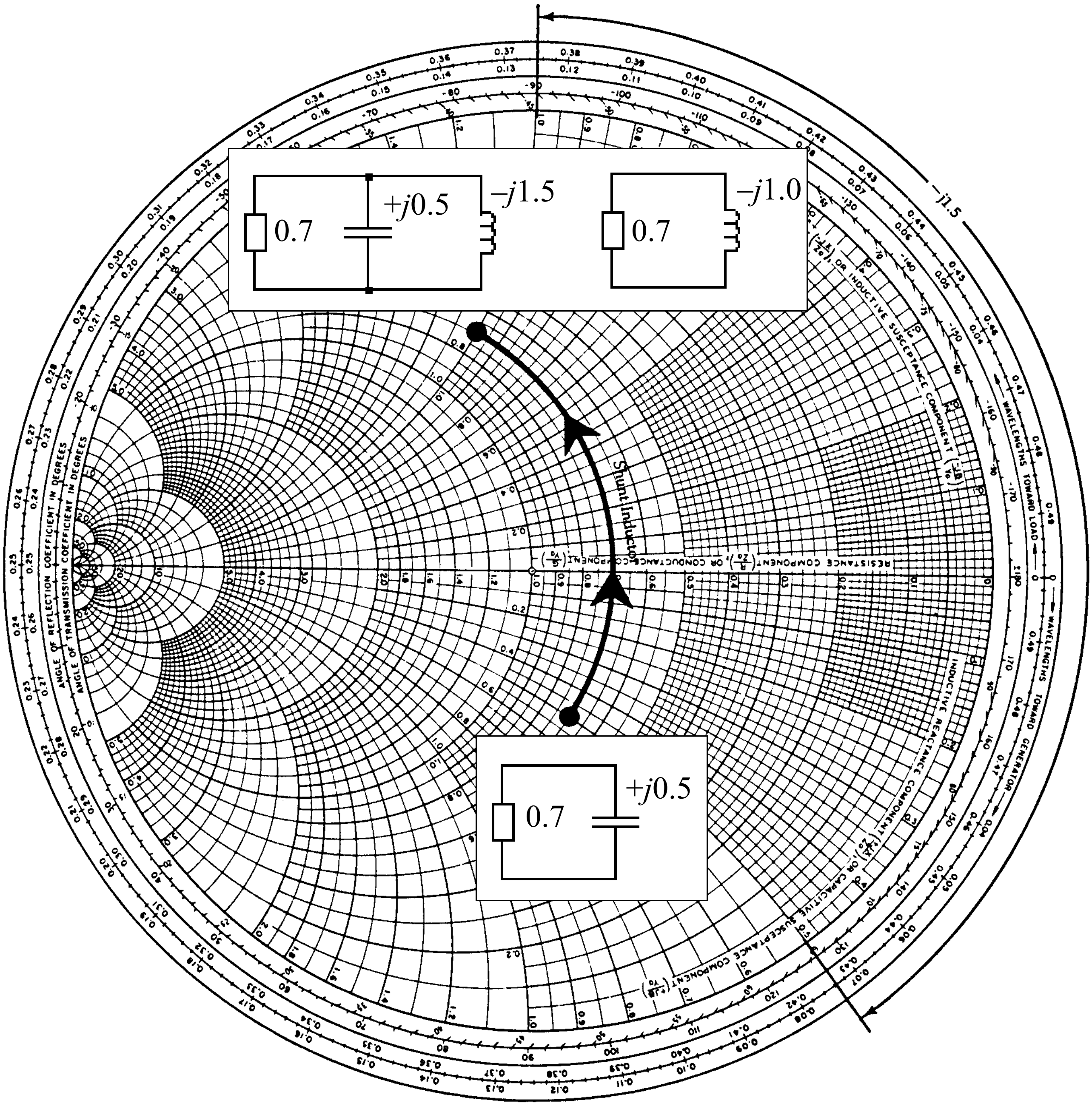


Fig 3-32: *Addition of a shunt inductor.*

SUMMARY OF SMITH CHART MANIPULATION

Fig. 3-32 presents a summary of the effect of the addition of components on a Smith Chart.

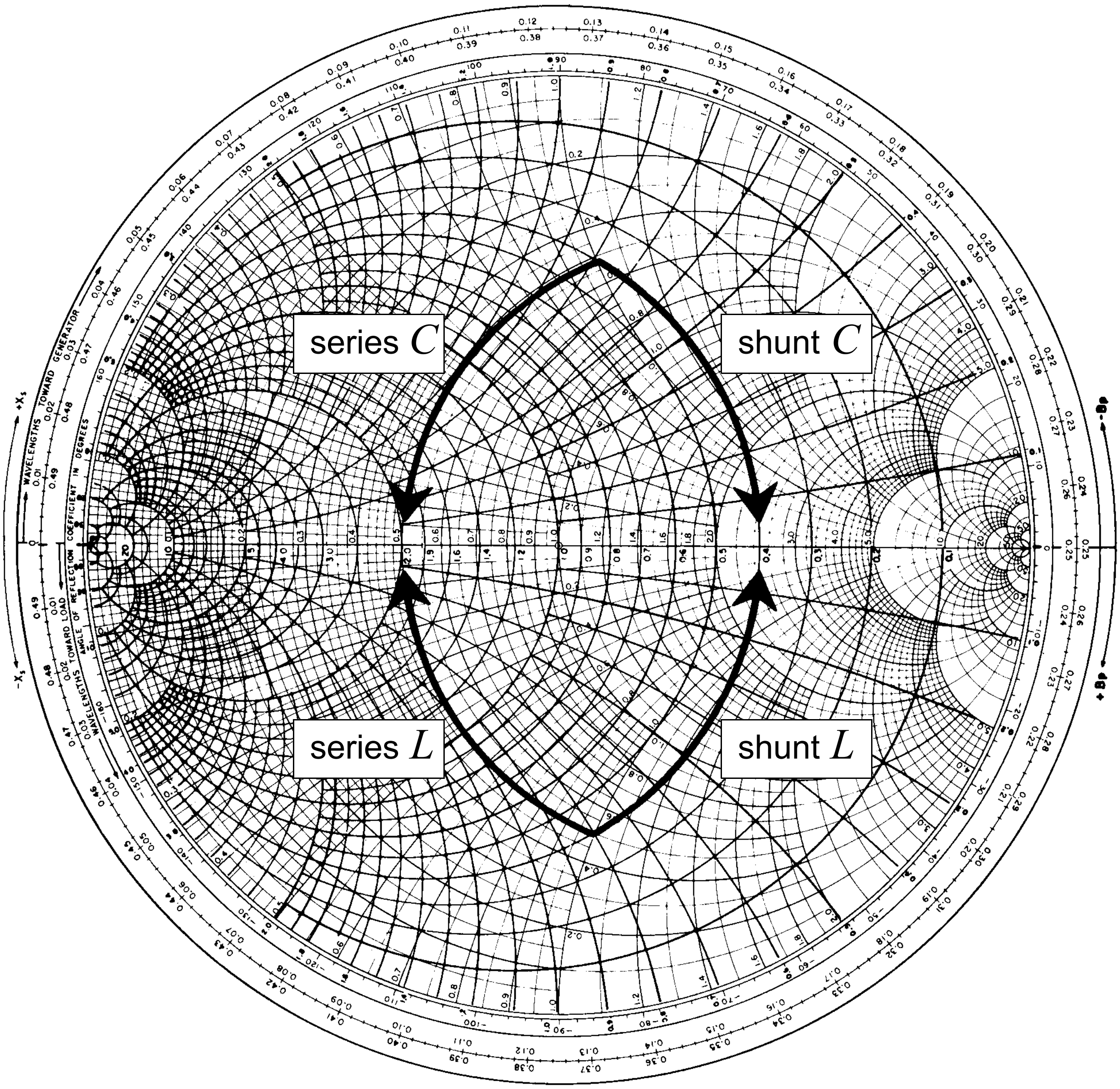


Fig 3-33: *Summary of the manipulation of components on the Smith Chart.*

EXAMPLE 1 (1/2)

Design a two-element circuit to impedance-match a source impedance $Z_S = 25 - j15$ to a load impedance $Z_L = 100 - j25$ for a frequency of 60 MHz. The transfer function should be of type low-pass.
The impedance that must be seen by the source is its complex conjugate, $Z_S{}^* = 25 + j15$. Thus, we must transform the load impedance to an impedance $Z_S{}^*$. We choose to normalize the impedance to $R_0 = 50\Omega$, so: $z_S{}^* = 0.5 + j0.3$ and $z_L = 2 - j0.5$. These two normalized impedances are represented respectively at point A (load) and at point C (source). We must link these two points by introducing series and parallel elements. The constraint of needing a low-pass characteristic, forces us to have a series inductor combined with a parallel capacitor. The only way to connect point A to point C while satisfying this demand is represented in Fig. 3-34. The arc AB corresponds to a shunt capacitor with normalized susceptance $+b = 0.73$. The arc BC corresponds to a series inductor with normalized reactance $+x = 1.2$. We find the values of the components by denormalizing according to the following equations:

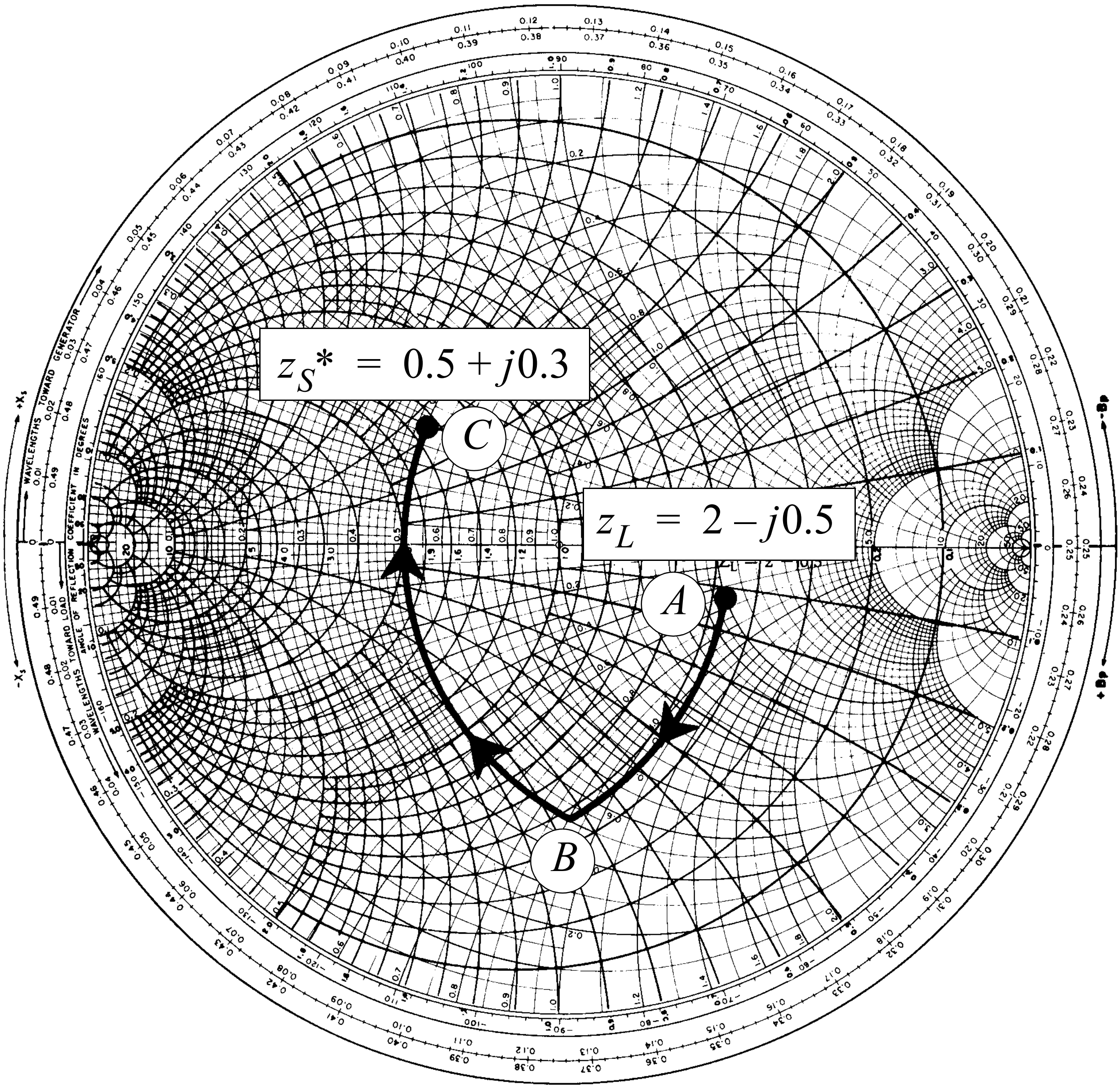
$$\begin{array}{ll} \textit{series:} & \textit{parallel:} \\ C = \dfrac{1}{\omega x R_0} & C = \dfrac{b}{\omega R_0} \\ L = \dfrac{x R_0}{\omega} & L = \dfrac{R_0}{\omega b} \end{array} \tag{3.18}$$

from which:

$$C = \frac{b}{\omega R_0} = \frac{0.73}{2\pi 60\times 10^6 \times 50} = 38.7pF$$

$$L = \frac{xR_0}{\omega} = \frac{1.2 \times 50}{2\pi 60\times 10^6} = 159nH$$

EXAMPLE 1 (2/2)



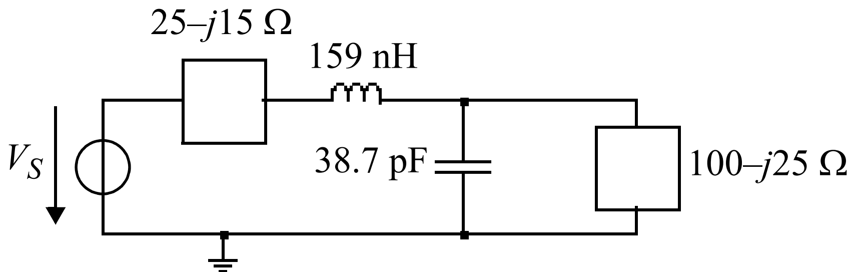


Fig 3-34: *Example of two-element impedance matching.*

CONSTANT-Q ARCS

We have seen that when matching networks of more than two elements, it is possible to choose the quality factor of the circuit. Fig. 3-35 represents the set of points with quality factor 5. These are situated on two arcs. The higher the quality factor, the more the arcs approach the circumference of the exterior circle representing an infinite quality factor.

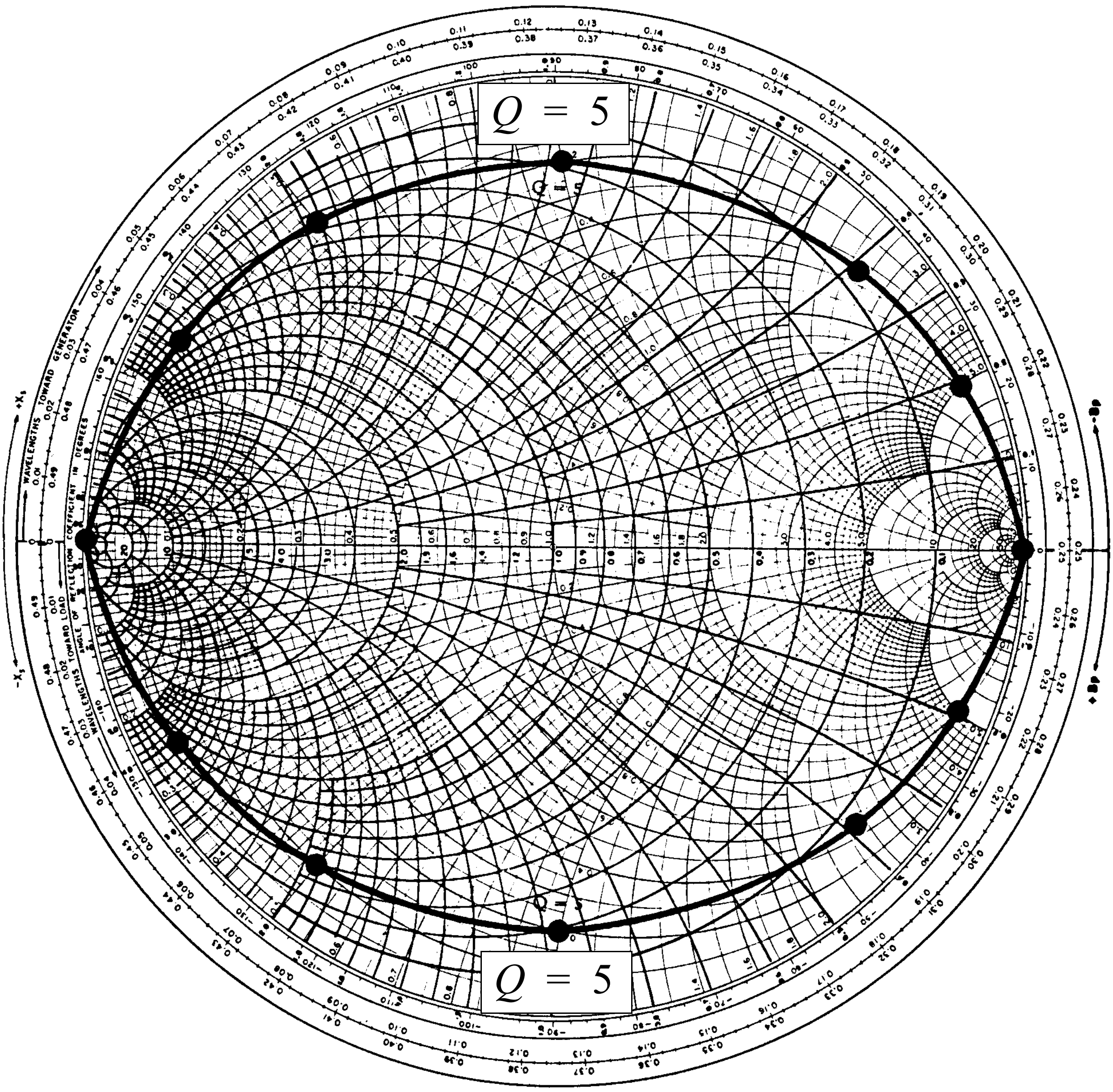


Fig 3-35: *Constant-Q arcs.*

THREE-ELEMENT NETWORKS

The design procedure using the Smith Chart for three-element matching circuits is as follows:

1) Draw the arcs corresponding to the specified quality factor;
2) Plot the normalized load impedance and the normalized source impedance;
3) Determine which of the terminating resistors will set the quality factor of the circuit: the smaller for T networks, and the larger for Π networks;
4) For T networks:

   $R_S > R_L$: from the load, move along a constant resistance circle to the intersection with the constant-Q arc. This arc will determine the value of the first element. Reach the point $z_s{}^*$ by first adding a shunt element and then a series element;

   $R_S < R_L$: find the intersection I of the constant-R circle of the source, with the constant-Q arc. Reach the point I from the load by using two elements: first a series element followed by a shunt element. Reach the point $z_s{}^*$ by moving around the constant-R circle with the help of another series element.
5) For Π networks:

   $R_S > R_L$: find the intersection I of the constant conductance circle of the source with the constant-Q arc. Leave from the load towards the point I first with a shunt element followed by a series element. Go toward the point $z_s{}^*$ on the constant-G circle by using another shunt element;

   $R_S < R_L$: leave from the load on the constant-G circle until reaching the intersection with the constant-Q arc. The length of this arc determines the value of the first shunt element. Go to the point $z_s{}^*$ by adding first a series element, followed by a shunt element.

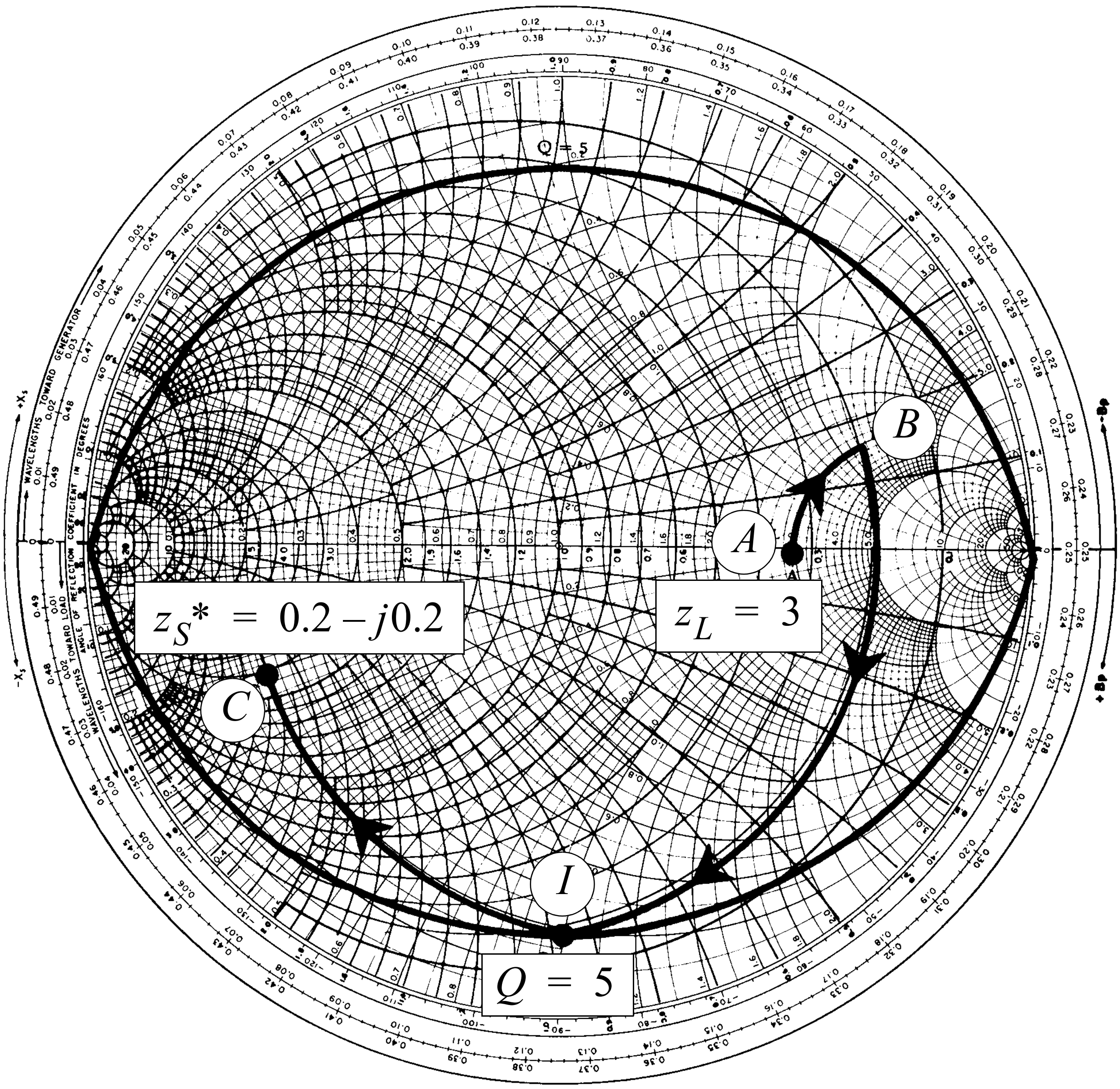
EXAMPLE 2 (1/2)

We would like to design a T network to impedance-match a source $Z_S = (15 + j15)\Omega$ with a load impedance $Z_L = 225\Omega$ for a frequency of 30 MHz and a quality factor of 5.
We normalize with $R_0 = 75\Omega$ and find $z_S{}^* = 0.2 - j0.2$ and $z_L = 3$. Since we want a T network, in this case it is the source termination that determines the quality factor. Following the procedure for $R_S < R_L$, it is first necessary to determine the intersection I of the constant-R circle that passes through $z_S{}^*$ and the constant-Q arc. Then we must leave from the load to go to this point I, first with a series inductor L_3 with reactance $x_3 = 2.5$ and a shunt capacitor C_2 with susceptance $b_2 = 1.15$. Then we move around the constant-R circle with a series inductor L_1 with reactance $x_1 = 0.8$. We calculate the values of the elements according to:

$$\begin{aligned} L_3 &= \frac{x_3 R_0}{\omega} = \frac{2.5 \times 75}{2\pi 30 \times 10^6} = 995nH \\ C_2 &= \frac{b_2}{\omega R_0} = \frac{1.15}{2\pi 30 \times 10^6 \times 75} = 81pF \\ L_1 &= \frac{x_1 R_0}{\omega} = \frac{0.8 \times 75}{2\pi 30 \times 10^6} = 318nH \end{aligned} \tag{3.19}$$

The resulting circuit and the design process are shown in Fig. 3-36.

EXAMPLE 2 (2/2)



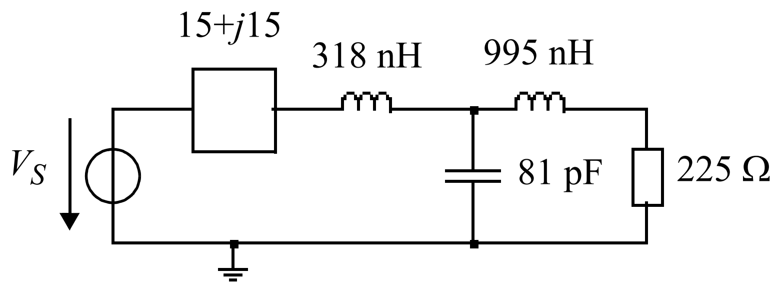


Fig 3-36: *Example of three-element impedance matching.*